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The Evaluation of Reliability Indices in Water Distribution Networks under Pipe Failure Condition

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Abstract

In this research, reliability indicators of water distribution networks are evaluated under pipe failure conditions. The case studies include two benchmark and one real-life water distribution networks in Iran with more hydraulic constraints. Some important reliability indicators are presented such as resilience index, network resilience, modified resilience index and minimum surplus head index. GANetXL is used to do one-objective and two-objective optimization of the previously mentioned water distribution networks in order to not only minimize the cost, but also maximize the reliability indicators. Moreover, the results of a statistical analysis for each pipe is used to determine the sensitive pipes that are of the most failure probability. GANetXL is an optimization tool in Excel environment and works based on Genetic Algorithm. GANetXL has the capability of being linked to EPANET (Hydraulic simulation software). The results obtained clearly show that network resilience index is of poor performance when compared with the other indexes under pipe failure conditions, especially in real-life networks that include small pipe diameters. It was also showed that if a water distribution network was optimized only in terms of cost, there would be an unacceptable pressure drop at some nodes in case of pipe failure.

Keywords: GANetXL, Optimization, Pipe Reliability, Resiliency, Water distribution Network

1 Introduction

Water distribution networks (WDNs) are designed to provide users with a minimum acceptable level of supply, in terms of pressure, availability, and water quality at all times under a range of operating conditions. Nowadays, WDNs have become

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complex and need huge investments in construction and maintenance. As a result, there is an avid desire to improve their efficiency through minimizing their cost and maximizing their benefit.

Optimal WDN design is a computationally complex problem because of its non-linear nature and the constraints involved. Therefore, finding the globally optimal solution is difficult if we use optimization methods as the non-linearity is significant. A majority of the previous models have utilized different optimization algorithms for ordinary benchmark networks meaning that optimal design problems have turned out to be more of mathematical or computational challenges rather than civil engineering ones.

In the last three decades, several researchers have broadly studied the design optimization problem of WDNs. The problems have been solved using linear, non-linear and various meta-heuristic methods. Linear and non-linear methods were predominantly used in the period 1960–1990(Jacoby 1968, Watanatada 1973, Alperovits and Shamir 1977, Quindry, Liebman et al. 1981, Lansey and Mays 1989, Fujiwara and Khang 1990). Linear methods applied to nonlinear problems have not resulted in optimal solutions. The non-linear methods did not necessarily yield a global optimum, and the final solution depended on the initial solution used as a starting point for the search procedure (Piratla 2016). In addition, the use of discrete variables, specific-size pipe diameters, limits the quality of the optimal solution obtained. These limitations led to the employment of meta-heuristics that use stochastic optimization methods.

Murphy and Simpson were the first researchers who used a simple Genetic Algorithm (GA) to optimally design water distribution systems. This model was applied to determine the least cost combination of pipe diameters and rehabilitation actions (Murphy and Simpson 1992). GA has been integrated with hydraulics simulator to optimize the solutions by many researchers (Simpson, Dandy et al. 1994, Simpson and Goldberg 1994, Savic and Walters 1997, Lippai, Heaney et al. 1999, Neelakantan, Suribabu et al. 2008). (Vasan and Simonovic 2010) recently applied a differential evolutionary algorithm (DE), an improved GA. The major difference between GA and DE is that GA relies on crossover, a mechanism of probabilistic exchange of information among solutions to create better solutions, while DE uses mutation as the primary search mechanism. DE uses a uniform crossover that can take child vector parameters from one parent more often than from the other one. It is said that GA most of the times succeed in finding the global optimum or at least arriving at somewhere very close to it. More importantly, GA is capable of handling discrete optimization (as pipe diameters are discrete) (Savic and Walters 1997).

Many other optimization algorithms have been used in the optimal design of water distribution systems (Tayfur 2017). (Loganathan, Greene et al. 1995) and (Cunha and Sousa 1999) applied simulated annealing for optimal design of water distribution systems. (Geem, Kim et al. 2002) developed a harmony search optimization approach to solve network design problems while (Eusuff and Lansey 2003) developed the shuffled frog leaping algorithm. (Maier, Simpson et al. 2003) applied the ant colony optimization approach and improved GA both in terms of computational efficiency and its ability to find nearly optimal solutions. (Baños, Gil et al. 2007) analyzed the performance of memetic algorithms for optimal design of looped water distribution systems and demonstrated that it works well for problems of large scale. (Mohan and Babu 2009) proposed to use a heuristic based approach called heuristics-based algorithm (HBA) to identify the least cost combination of pipe diameters. They demonstrated that the HBA is capable of identifying the least cost combination of pipe diameters with fewer numbers of evaluations. (Moghaddam, Alizadeh et al. 2018) applied a Simple Modified Particle Swarm Optimization (SMPSO) to minimize the cost of water distribution networks. SMPSO then used a novel factor to decrease the inertia weight of the algorithm in proportion with simulation time to facilitate both global and local search.

Objective function is important in optimizing the design of distribution systems. The main negative aspect of the single-objective constrained formulation is that it does not effectively set up a trade-off between cost and reliability/robustness of a design (Todini 2000). Reliability can be considered as the ability of providing an adequate supply under both usual and unusual conditions (Farmani, Savic et al. 2005), including demand uncertainty, pipe failure, etc. One of the most used reliability criteria is the concept of resilience index suggested by (Todini 2000),which is a measure of the ability of the network to handle failures and is related indirectly to system reliability. Several suggestions were made to modify the resilience index introduced by Todini (Prasad and Park 2004, Farmani, Savic et al. 2005, Jayaram and Srinivasan 2008, Reca, Martinez et al. 2008, Raad, Sinske et al. 2010, Baños, Reca et al. 2011, Greco, Di Nardo et al. 2012, Pandit and Crittenden 2012).

Literature review shows that stochastic models, particularly the GA types, give better results than linear and non-linear optimization models (Pandit and Crittenden 2012). Subsequently, a genetic algorithm technique is used in this research as a part of GANetXL(Savić, Bicik et al. 2011) , an add-in to Microsoft Excel. GANetXL is used to optimize two benchmark networks from literature (Two-loop and Hanoi water networks) in two different conditions including single-objective (cost) and two-objective (cost and reliability criteria) optimizations. Afterwards, the solutions obtained, as well as the performance of the proposed Resilience Index, Network Resilience, Modified Resilience Index and Minimum Surplus Head Index are discussed. Finally, as the results obtained for the benchmark networks are satisfactory, GANetXL is used to design a real-life water network in Iran in which there are more hydraulic constraints compared with the benchmark networks. There are a few applications of GANetXL in water systems, which include the development of a model for optimal management of groundwater contamination (Farmani, Savic et al. 2005, Farmani, Henriksen et al. 2009) and multi-objective optimization of water distribution systems (Piratla and Ariaratnam 2012, Mala-Jetmarova, Barton et al. 2015, Piratla 2016).

2 Material and Methods

2.1 Optimization Model for WDN Design

In this paper, WDNs are optimized with pipe diameters as decision variables. Cost is considered as the objective function that must be minimized [Eq. (1)] and the reliability criteria are modeled in the form of a two-objective function [Eq. (2)].

$$Min\ f_1 = \sum_{i=1}^{N} c_i D_i \times l_i \qquad (1)$$

$$Max\ f_2 = Reliability\ factor \qquad (2)$$

Where f_1 is network cost, f_2 is network reliability, c_i is cost for unit length of pipe with diameter, D_i length l_i and N is pipe numbers in the network.

2.2 Constraints

The constraints to the optimization problem are as follows:

1) Explicit system constraints such as conservation of mass of flow, conservation of energy and conservation of mass of constituent, which all are controlled by water network simulator software, EPANET (Rossman 2000, Mala-Jetmarova, Barton et al. 2015).

2) Implicit bound constraints, which include choosing pipe diameters from a commercially available set of discrete pipe sizes [Eq. (3)], minimum and maximum pressure at load nodes [Eq. (4)], minimum and maximum velocity in pipes [Eq. (5)].

$$D_i \in \{CD_k\} \quad \forall i \qquad k = 1,2,3,\dots,nc \qquad (3)$$

$$H_j^{min} \leq H_j \leq H_j^{max}, \qquad j = 1,2,\dots,nd \tag{4}$$

$$V_i^{min} \leq V_i \leq V_i^{max}, \qquad i = 1,2,\dots,np \tag{5}$$

where D_i = diameter of pipe i; CD_k= kth commercially available pipe size; nc = number of available pipe sizes; H_j = hydraulic-head available at node j; H_j^{min}= minimum hydraulic-head required at node j; H_j^{max}=maximum hydraulic-head at node j; nd = number of demand nodes; V_i^{min}= minimum velocity required at pipe i and V_i^{max}=maximum velocity at pipe i; np = number of pipes.

2.3 Reliability Indicators

A range of reliability criteria has been introduced to different degrees of complexity. Usually, these criteria give some suggestion of the ability of a WDN to handle changing conditions and are straightforward to analyze so are practical for optimization studies that compare the performance of network design. This section presents the definition of the key criteria and their derivatives as well as the advantages and disadvantages of them.

2.3.1 Resilience Index (I_r)

Todini's resilience index is a popular surrogate measure within the WDN research field (Todini 2000, Reca, Martinez et al. 2008, Atkinson, Farmani et al. 2014). It considers surplus hydraulic power as a proportion of available hydraulic power. The resilience index,I_r, is measured in the continuous range of [0-1] (for feasible solutions of $H_j^{min} \leq H_j$) and is formulated as below (Todini 2000):

$$I_r = \frac{\sum_{j=1}^{nn} q_j (H_j - H_j^{min})}{\sum_{k=1}^{nr} Q_k H_k + \sum_{i=1}^{np} P_i/\gamma - \sum_{j=1}^{nn} q_j H_j^{min}} \tag{6}$$

Where nn is the number of supply and demand nodes; nr is the set of supply nodes (reservoir/emptying tanks); np denotes the number of pumps; H_j is the available head at supply node j; H_j^{min} represents the required head at supply node j; q_j is the demand at node j; $Q_k$is the supply at input node k; $H_k$is representative of head associated with the input node k; P_i is the power of pump i; and finally γ is the specific weight of water. Maximization of the resilience index improves the ability of a pipeline network in encountering failure conditions.

2.3.2 Network Resilience (I_n)

Prasad and Park (2004) introduced another reliability measure called network resilience (I_n), which incorporates the effects of both surplus power and reliable loops. Reliable loops can be ensured if the pipes connected to the same node do not vary greatly in diameter. If D_{1j}, D_{2j}, ..., D_{npj} (where $D_{1j} \geq D_{2j} \geq \cdots \geq D_{npj}$) are the diameters of the np pipes connected to node j, then uniformity of that node is given by Eq. 7,

$$C_j = \frac{\sum_{i=1}^{np} D_{ij}}{np \times \max D_{ij}} \tag{7}$$

where npj is the number of pipes connected to node j. The value of C_j = 1 if the diameter of the pipes connected to the same node are the same; and C_j<1 if the pipes connected to a node have different diameters. For nodes connected to only one pipe, the value of C_j is taken to be one.

$$I_n = \frac{\sum_{j=1}^{nn} C_j q_j (H_j - H_j^{min})}{\sum_{k=1}^{nr} Q_k H_k + \sum_{i=1}^{np} {}^{P_i}/_{\gamma} - \sum_{j=1}^{nn} q_j H_j^{min}} \qquad (8)$$

Theoretically, the value of network resilience may vary between 0 and 1. However, for real-world systems it never attains a value of 1, since imposing the same diameter to all pipes in a network need not always provide a Pareto-optimal solution in Cost-I_n space, as I_n is a measure of the combined effect of surplus power and nodal uniformity.

2.3.3 Modified Resilience Index (MRI)

Jayaram and Srinivasan(Jayaram and Srinivasan 2008) proposed a modified resilience index (MRI), which theoretically overcomes the drawback of Todini's resilience index when evaluating networks with multiple sources. In contrast to Todini's resilience index, the value of the modified resilience index is directly proportional to the total surplus power at the demand nodes. Eq. (9) describes MRI, which only considers the solutions with pressures equal to or higher than that required in all nodes. While Todini's I_r and Prasad's I_n take values up to a maximum of 1, Jayaram's MRI can be greater than 1(Baños, Reca et al. 2011).

$$MRI = \frac{\sum_{j=1}^{nn} q_j H_j - H_j^{min}}{\sum_{j=1}^{nn} q_j H_j^{min}} \qquad (9)$$

2.3.4 Minimum Surplus Head Index (I_m)

In a WDN, minimum surplus head, I_m, is defined as the lowest nodal pressure difference between the minimum required and observed pressure, formulated as

$$I_m = min\{H_j - H_j^{min}\} \qquad j = 1,2,\dots,nn \qquad (10)$$

Maximization of the available surplus head at the most depressed node to some extent improves the reliability of a network (Prasad and Park 2004).

2.4 GANetXL

GANetXL is used as the optimization tool in this research. GANetXL has been developed by the Center for Water System of University of Exeter as an add-on in Microsoft Excel (Miri and Afshar 2014). It is a common optimization tool with spreadsheet-based interface for solving both single-objective and multi-objective optimization problems (Savić, Bicik et al. 2011). The primary advantage of GANetXL is its capability of easy integration with EPANET via Visual Basic. GANetXL incorporates GA for single-objective and NSGA-II for multi-objective optimizations (Deb, Pratap et al. 2002). In addition, it has the capability to apply penalty functions. GANetXL is well suited for solving multi-objective optimization problems (Mala-Jetmarova, Barton et al. 2014).

In this paper GANetXL is employed in two steps: in the first step for single-objective optimization based on GA and the second step for two-objective optimization based on NSGA-II. GA and NSGA-II parameters such as population size, the number of generations, selection method, crossover and mutation operators, crossover and mutation probability and the type of algorithm were tested and reasonably well-performing parameters selected for final optimization runs. These parameters are presented in Table 1 for three example networks, which are described in the following sections. The crossover and mutation types are described in details in CWS (2011).

Table 1. Optimum GA and NSGA-II values for the three case studies in this paper

Parameter	Parameter values or method selected	
	Two-loop Network	**Hanoi and Real- life Network**
Algorithm (Only in single-objective mode)	Generalation	Generational Elitist
Population size	70	100
Number of generations	1000	1000
Selection Method	Roulette	Tourenmate
Crossover operator	Simple one point	Simple one point
Mutation operator	Simple by gene	Simple
Crossover probability	0.8	0.95
Mutation rate	0.01	0.7
Adaptive mutation	Yes	Yes

3. Results and Discussion

Three example applications are studied: the Two-loop (Alperovits and Shamir 1977), Hanoi (Fujiwara and Khang 1990) ,which are the benchmark networks, as well as a real-life case study in Iran.

3.1 Example 1: The Two - loop network

The Two-loop network, shown in figure. 1, was originally presented by Alperovits and Shamir (Alperovits and Shamir 1977). The network consists of 7 nodes and 8 pipes with two loops, and is fed by gravity from a reservoir with a 210 m fixed head. Nodal demands and elevations are given in Table 2. The pipes are all 1000 m long with the assumed Hazen–Williams coefficient of 130. The minimum pressure head requirement of the other nodes is 30 m above the nodal elevations. There are 14 commercial diameters to be selected whose costs and diameters are given in Table 3.

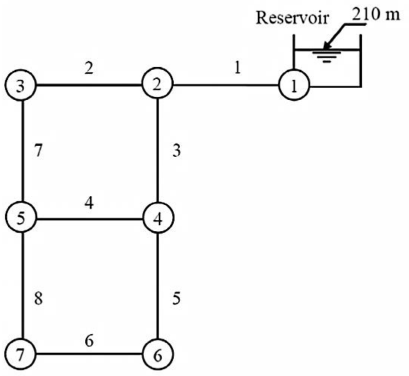

Fig. 1 The layout of Two-loop network.

Table 2. Node demands and elevations for Two-loop network

Node	Elevation (m)	Demand (m^3/h)

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2	150	100
3	160	100
4	155	120
5	150	270
6	165	330
7	160	200
Reservoir 1	210	-1120

Table 3. Pipe sizes and costs for Two-loop network

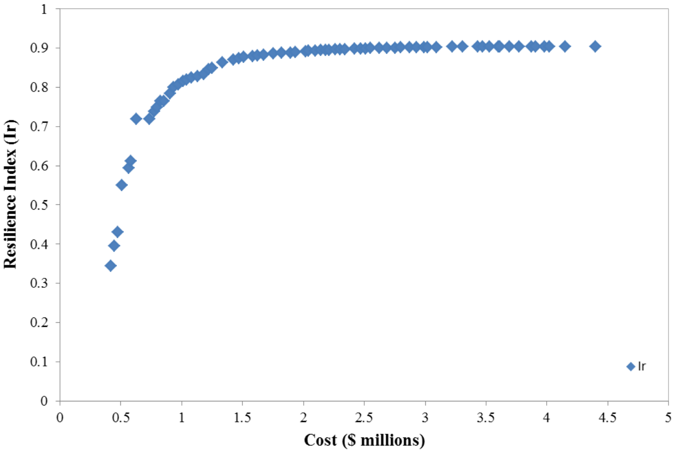
Pipe	Diameter (mm)	Cost ($/m)
1	25.4	2
2	50.8	5
3	76.2	8
4	101.6	11
5	152.4	16
6	203.2	23
7	254.0	32
8	304.8	50
9	355.6	60
10	406.4	90
11	457.2	130
12	508.0	170
13	558.8	300
14	609.6	550

In the first step, as a result of single-objective optimization of the Two-loop network using GA technique in GANetXL, the minimum cost obtained 419000$ with 35000 number of function evaluations (NFEs) which is the same to minimum costs obtained by GA (Savic and Walters 1997), Simulated Annealing (SA) (Cunha and Sousa 1999), Shuffled frog leaping Algorithm (SFLA) (Eusuff and Lansey 2003), Harmony Search (HS) (Geem 2009)and Scatter search (SS)(Lin, Liu et al. 2007) with 250000, 25000, 11323, 5000 and 3215 NFEs, respectively.

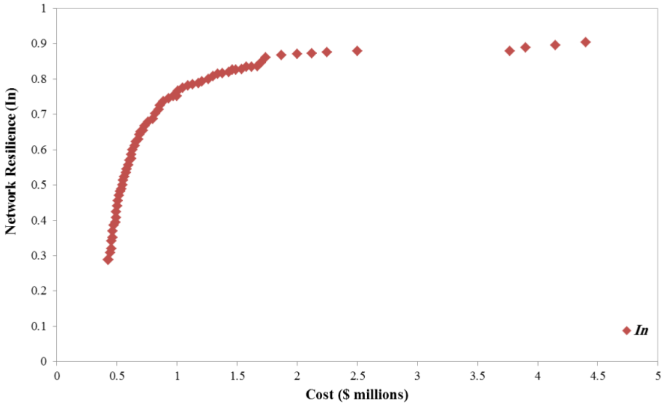
As a result, minimum cost is 419000$ for one-objective optimization of this network using GANetXL after 1000 generations that is equal with minimum costs obtained by GA, Simulated Annealing (SA), Shuffled frog leaping Algorithm (SFLA) Harmony Search (HS) and Scatter search (SS)(Savic and Walters 1997, Cunha and Sousa 1999, Geem, Kim et al. 2002, Eusuff and Lansey 2003, Geem 2009).

In the second step, figure 2 (a-d) shows the obtained Pareto front for two-objective optimization of two-loop network using NSGA-II in GANetXL considering I_r, I_n, MRI and I_m as the second objective function, respectively. All of the solutions

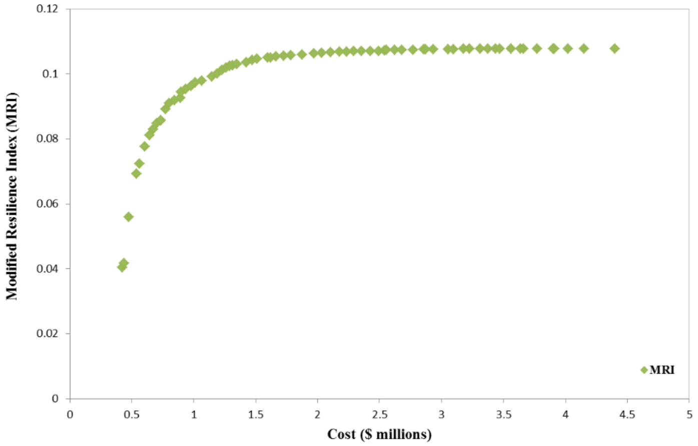
205 in this Pareto front are feasible (and all the network constraints are satisfied). As it is observed the cost changes in the range of [0.424×10^6 - 4.400×10^6] \$ and I_r, I_n, MRI and I_m criteria changes in the ranges [0.338-0.903], [0.287-0.903], [0.040-0.107] and [0.122-12.856], respectively. In the cost range of [0.424×10^6-1×10^6]\$, Cost-I_n Pareto front shows more and varied solutions, in comparison to other graphs. However, with increase in cost, non-dominated solutions decreases and the current continuity in Pareto front disappears while Cost-I_r and Cost-MRI Pareto fronts have better performance. In Cost-I_m graph
210 the variety of obtained solutions in the lower and upper bound of Pareto front is lower than other graphs.



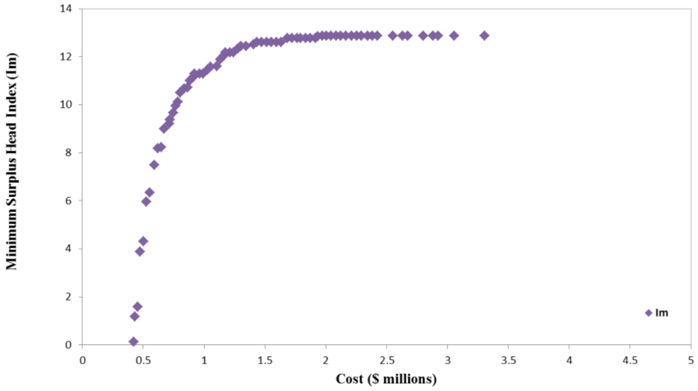
(a)



(b)



(c)



(d)

Fig. 2 Pareto front of two-objective function optimization of the Two-loop network, (a) Cost-I_r, (b) Cost-I_n, (c) Cost-MRI, (d) Cost-I_m

Figure 3 shows the surplus pressure of the minimum pressure head requirement in the nodes of Two-loop network for solutions with maximum reliability criteria and minimum cost. As it is observed, the surplus pressure of the nodes in the solutions with minimum cost is lower than the solutions of maximum reliability criteria (I_r, I_n, MRI and I_m). Also, the design based on single-objective function (minimum cost), surplus pressure is closer to the minimum allowed pressure in nodes 3, 6, and 7, showing that these nodes are the critical nodes of the network. As a result, if the two-loop network was designed only based on minimum cost, in critical periods such as pipe failures, there would be problems issues at these nodes.

Reliability evaluation should be analyzed under all feasible extreme conditions. Failure of multiple pipes as well as the failure of the reservoir connection line during a firefighting event and/or power or pumping station failures should be evaluated simultaneously. Although an infinite number of failure scenarios are likely, the probability of simultaneous failures in multiple pipes is too low (Tabesh, Tanyimboh et al. 2001). Pipe failures independency can be assumed (Su, Mays et al. 1987) and any likely dependency will be negative. For example, if a pipe failure occurs in the network, the pressure will decrease, and consequently the probability of another pipe failure will decrease as well. However, in case the system is a large-scale WDN, the influence of pressure might not be significant. Other pipe failure reasons, such as damages or traffic loadings, may lead to pipe failures that are completely independent events (Shafiqul Islam, Sadiq et al. 2013).

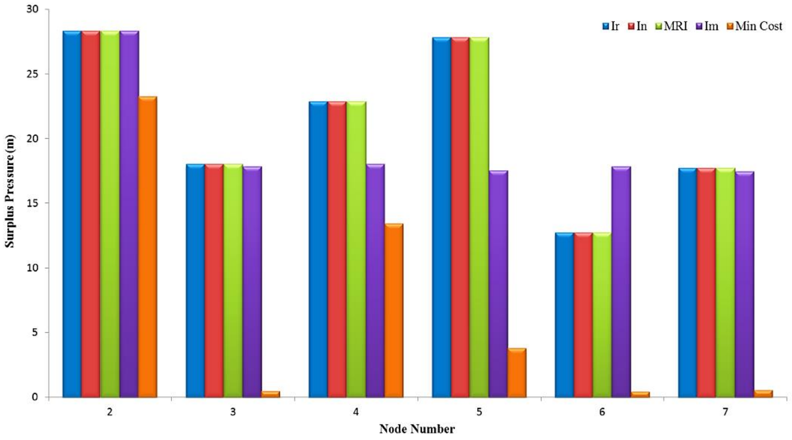


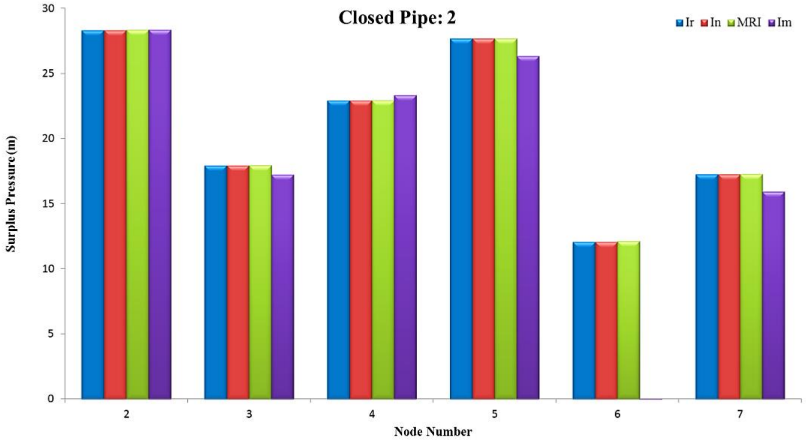
Fig. 3 Surplus pressure of nodes in two-loop network for solutions of maximum reliability criteria and minimum cost

In this paper, to evaluate reliability of the candidate solutions of maximum I_r, I_n, MRI and I_m criteria, the nodal pressures have been investigated under pipe failure conditions. Table 4 presents the statistical parameters of each pipe in two-loop network under different runs of single-objective optimizations when the objective function is to minimize cost. This table helps the designers to recognize critical and sensitive pipes that have the most probability of failure in the network. For example, maximum and minimum diameters that are allocated to pipe 1 in different runs of GANetXL are 24 and 18 inch, respectively.

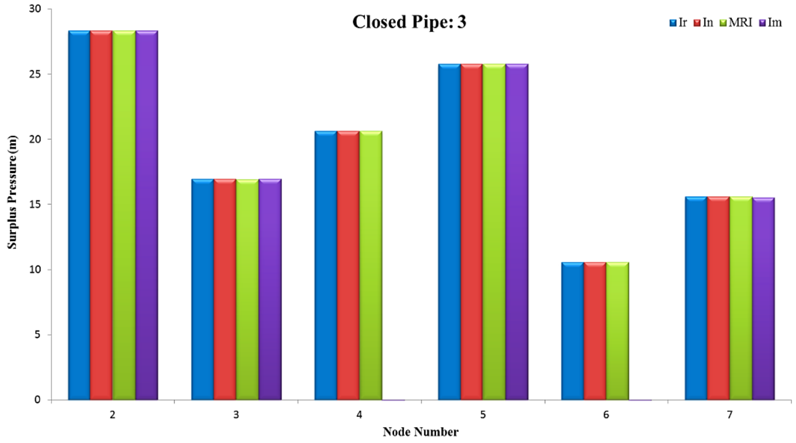
Table 4. Statistical parameters for diameters obtained for each pipe of two-loop network

Pipe numb	Maximum (m	Minimum (m	Average (m	ST. DEV	Variance	C.V.
1	609.6	457.2	459.35	17.96	322.52	0.04
2	304.8	152.4	248.28	**26.35**	694.18	**0.11**
3	457.2	406.4	452.91	**14.13**	199.65	**0.03**
4	254	101.6	208.21	41.03	1683.23	0.20
5	609.6	406.4	469.36	**37.62**	1414.97	**0.08**
6	508	101.6	251.85	57.47	3302.97	0.23
7	558.8	76.2	217.87	63.47	4028.38	0.29
8	304.8	25.4	68.69	58.51	3423.79	0.85

According to Table 4, pipe numbers 2, 3 and 5 that have minimum standard deviations and variation coefficients are chosen for failure analysis. Pipe 1 belongs to a water transmission line from the reservoir to the network that is important during network operation. If a failure is considered in this pipe, then the network will be unreliable. That is why this pipe is not taken into account for failure analysis. Figure 4 shows the performance of solutions with maximum reliability criteria under the failure of pipes 2, 3 and 5.



(a)



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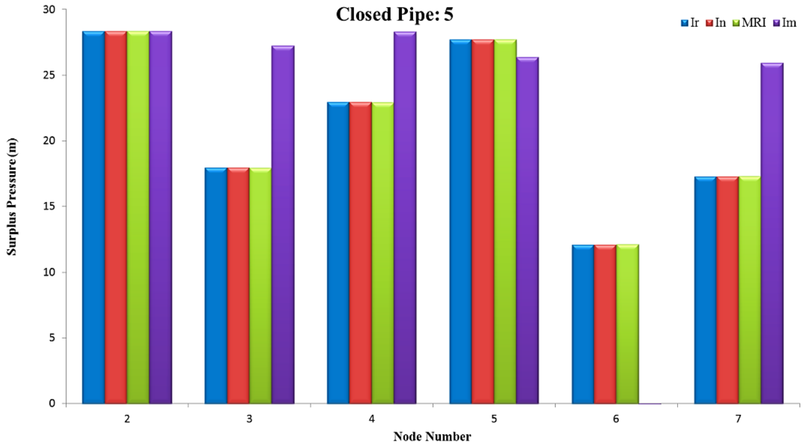

(c)

Fig. 4 Surplus pressure of nodes in two-loop network for solution with maximum reliability criteria under failure of pipes No. (a) 2, (b) 3 and (c) 5

Figure 4 shows that, nodeNo. 6 encounters with a serious pressure loss with failure in pipe No. 2, 3 and 5 in represented solutions by I_m criterion. In represented solutions based on I_r , I_n and MRI for all the pipes of the network the diameter was 609.6 mm while in the obtained solution with maximum I_m , the diameter of pipes No. 4 and 6 was 25.4 mm and other pipes were 609.6 mm. Consequently, I_m criterion is of lower performance than any other criterion under pipe failure condition.

3.2 Example 2: The Hanoi network

The Hanoi network in Vietnam (Figure. 5), first presented by Fujiwara and Khang, is a new design as all new pipes are to be selected. The network consists of 32 nodes and 34 pipes organized in three loops. The system is gravity fed by a single reservoir. The network details are given in Table 5. The minimum required pressure head for all nodes is 30 m and the elevation for all nodes is zero. There are six available pipe diameters to be selected for each new pipe and the pipe cost per meter for the six available pipe diameters are listed in Table 6.

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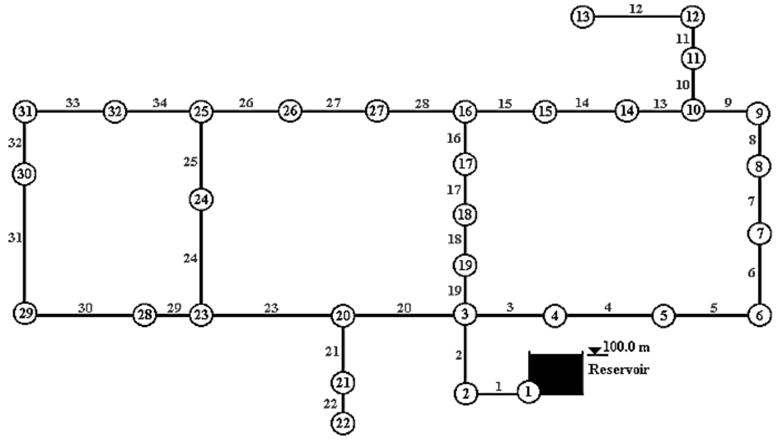


Fig. 5 Layout of Hanoi network

Table 5. Network data for the Hanoi network

Node data		Pipe data	
Node	**Demand (m**	**Pipe**	**Length (m**
1	-19940	**1**	100
2	890	**2**	1350
3	850	**3**	900
4	130	**4**	1150
5	725	**5**	1450
6	1005	**6**	450
7	1350	**7**	850
8	550	**8**	850
9	525	**9**	800
10	525	**10**	950
11	500	**11**	1200
12	560	**12**	3500
13	940	**13**	800
14	615	**14**	500
15	280	**15**	550
16	310	**16**	2730
17	865	**17**	1750
18	1345	**18**	800
19	60	**19**	400
20	1275	**20**	2200
21	930	**21**	1500

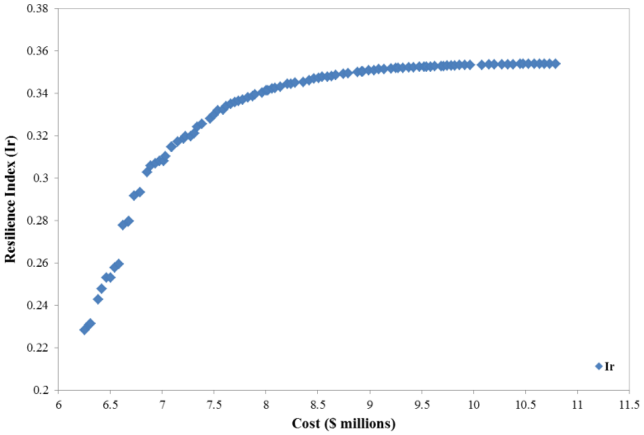
22	485	**22**	500
23	1045	**23**	2650
24	820	**24**	1230
25	170	**25**	1300
26	900	**26**	850
27	370	**27**	300
28	290	**28**	750
29	360	**29**	1500
30	360	**30**	2000
31	105	**31**	1600
32	805	**32**	150
		33	860
		34	950

Table 6. Pipe sizes and costs for Hanoi network

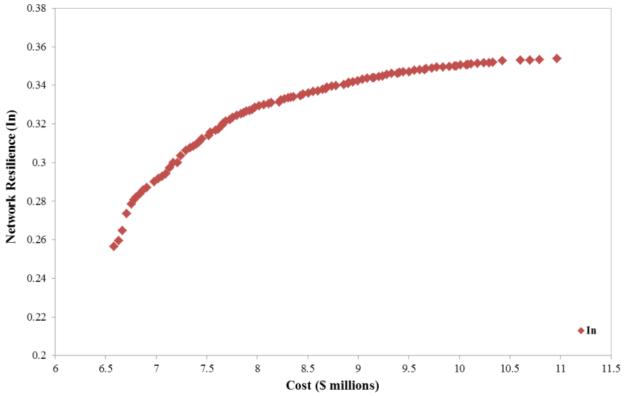
Pipe	Diameter (mm)	Cost ($/m)
1	304.8	45.726
2	406.4	70.400
3	508.0	98.378
4	609.6	129.333
5	762.0	180.748
6	1016.0	278.280

In the first step, as a result of single-objective optimization, GA method in GANetXL obtained a minimum cost of 6.097×10^6\$ with 100000 NFEs for this network while in the previous researches the methods of GA(Savic and Walters 1997), Ant Colony Optimization (ACO)(Zecchin, Simpson et al. 2006), and Shuffled Complex Evolution (SCE) Liong and Atiquzzaman (Atiquzzaman and Liong 2004)reported costs of 6.195, 6.134 and 6.22 million$ with 1000000, 25402 and 85571 NFEs, respectively.

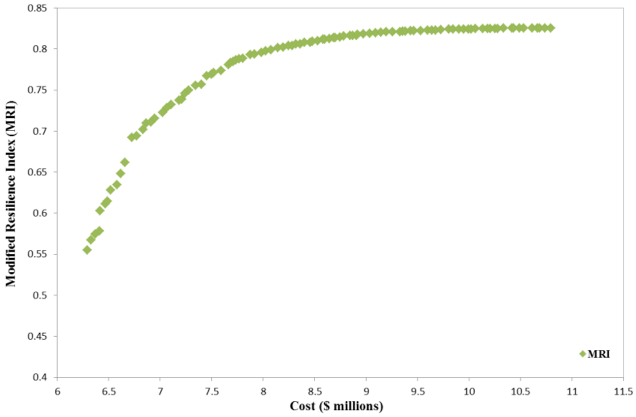
In the second step, figure 6. (a-d) shows non-dominated solutions of Hanoi network which calculated by NSGA-II considering minimum cost versus maximum reliability criteria and all of the solutions in the Pareto front is feasible. As it is observed in figure 7 minimum values of I_r, I_n, MRI and I_m are 0.228, 0.256, 0.555 and 0.090 and maximum values are 0.353, 0.353, 0.825 and 19.916, respectively. Cost values change in a range of [6.251×10^6-10.791×10^6]\$ for Cost-I_r and in [6.584×10^6-10.969×10^6]\$ Cost-I_n space that the increase in Cost-I_n to Cost-I_r is due C_j factor in formula [Eq. (8)] which cause uniformity diameters in the design phase. In this example monotony and variety of represented solutions are observed in all Pareto fronts, the reason can be found in the increase of the network size and possible solutions for network design.


Resilience Index (Ir)
0.38
0.36
0.34
0.32
0.3
0.28
0.26
0.24
0.22
0.2
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
Cost ($ millions)
◆ Ir

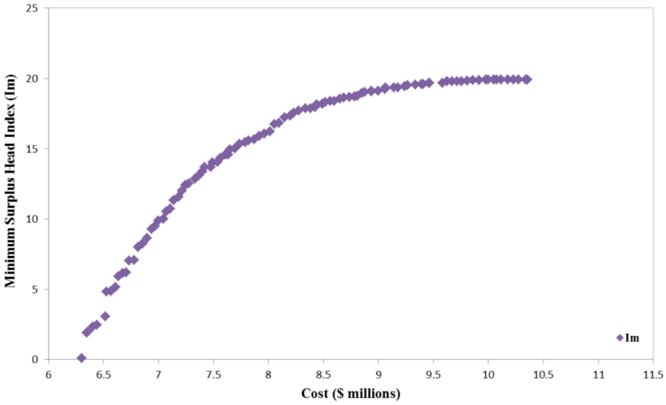
(a)


Network Resilience (In)
0.38
0.36
0.34
0.32
0.3
0.28
0.26
0.24
0.22
0.2
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
Cost ($ millions)
◆ In

(b)


Modified Resilience Index (MRI)
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
Cost ($ millions)
◆ MRI

(c)



(d)

Fig. 6 Pareto front of two-objective function optimization of the Hanoi network, (a) Cost-I_r, (b) Cost-I_n, (c) Cost-MRI, (d) Cost-I_m

Figure 7 shows the surplus pressure in comparison with minimum allowed pressure in the nodes of the Hanoi network for solutions of maximum reliability criteria and minimum cost. In the cost-based optimization, surplus pressure in nodes No. 13, 30 and 31 is less than 1 m which shows that these nodes are the most critical ones of this network. I_r, I_n and MRI criteria have similar performance for all the nodes, but I_m criterion determinates more surplus pressure for most of the nodes than other criteria in this network unlike the two-loop network.

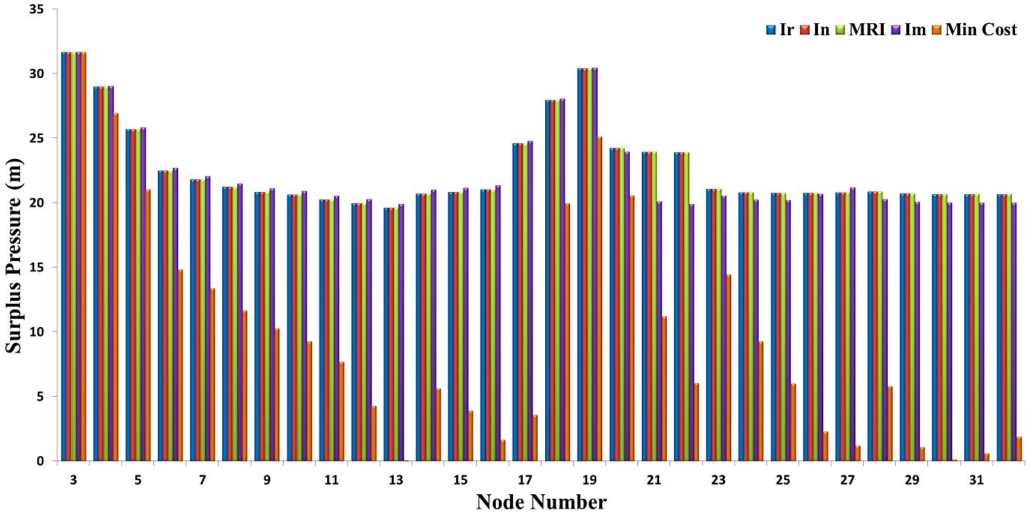


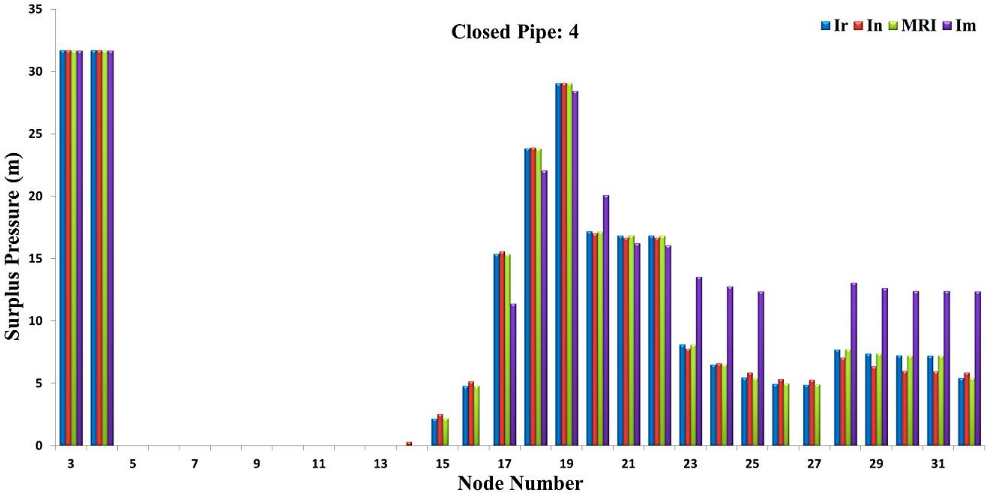
Fig. 7 Nodal surplus pressure of Hanoi network for solutions of maximum reliability criteria and minimum cost

Table 7 shows the statistical parameters for each pipe of Hanoi network due to different runs of single-objective optimizations by GANetXL. According to this table, Pipes No. 4, 5, 6 and 20 that have standard deviation and variation coefficient equal to zero have been chosen for reliability evaluation when there is a failure in the network.

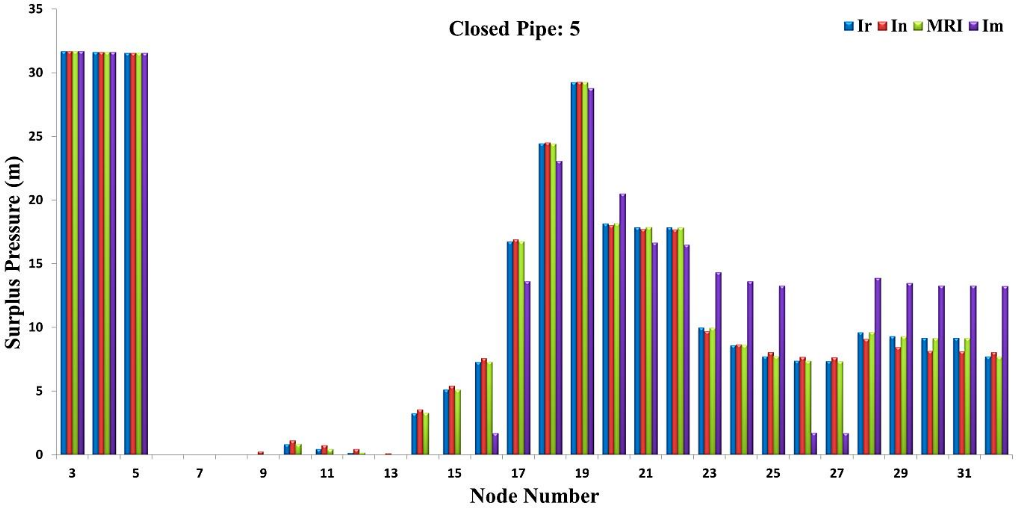
Table 7. Statistical parameters for diameters obtained for each pipe of Hanoi network

Pipe numbe	Maximum (m	Minimum (n	Average (n	ST. DEV	Variance	C.V.
1	1016	762	1005.84	49.77	2477.41	0.05
2	1016	762	1010.92	35.56	1264.51	0.04
3	1016	508	1005.84	61.38	3767.73	0.06
4	1016	1016	1016	**0**	0	**0**
5	1016	1016	1016	**0**	0	**0**
6	1016	1016	1016	**0**	0	**0**
7	1016	762	1013.46	25.27	638.71	0.02
8	1016	762	1005.84	49.77	2477.41	0.05
9	1016	508	1008.38	56.28	3167.74	0.06
10	1016	508	889	136.78	18709.64	0.15
11	762	304.8	608.584	47.64	2269.93	0.08
12	762	508	611.124	31.69	1004.13	0.05
13	1016	406.4	495.808	67.82	4599.73	0.14
14	1016	304.8	477.52	107.52	11561.27	0.23
15	762	304.8	387.604	146.47	21453.12	0.38
16	1016	304.8	341.376	110.85	12287.98	0.32
17	508	406.4	447.04	49.77	2477.41	0.11
18	1016	508	662.432	137.61	18937.77	0.21
19	762	406.4	511.048	43.59	1900.38	0.09
20	1016	1016	1016	**0**	0	**0**
21	1016	406.4	510.032	53.72	2886.19	0.11
22	1016	304.8	399.288	210.32	44233.20	0.53
23	1016	762	1013.46	25.27	638.71	0.02
24	1016	609.6	824.484	120.18	14444.10	0.15
25	1016	609.6	850.392	126.00	15877.13	0.15
26	1016	406.4	541.528	117.66	13843.59	0.22
27	1016	304.8	414.528	220.58	48656.42	0.53
28	1016	304.8	369.824	135.95	18481.51	0.37
29	1016	304.8	504.952	102.06	10416.50	0.20
30	609.6	304.8	446.024	64.09	4107.35	0.14
31	609.6	304.8	346.456	78.82	6213.15	0.23
32	1016	304.8	799.592	263.14	69244.76	0.33
33	1016	304.8	491.236	163.36	26686.66	0.33
34	762	406.4	514.604	40.42	1633.80	0.08

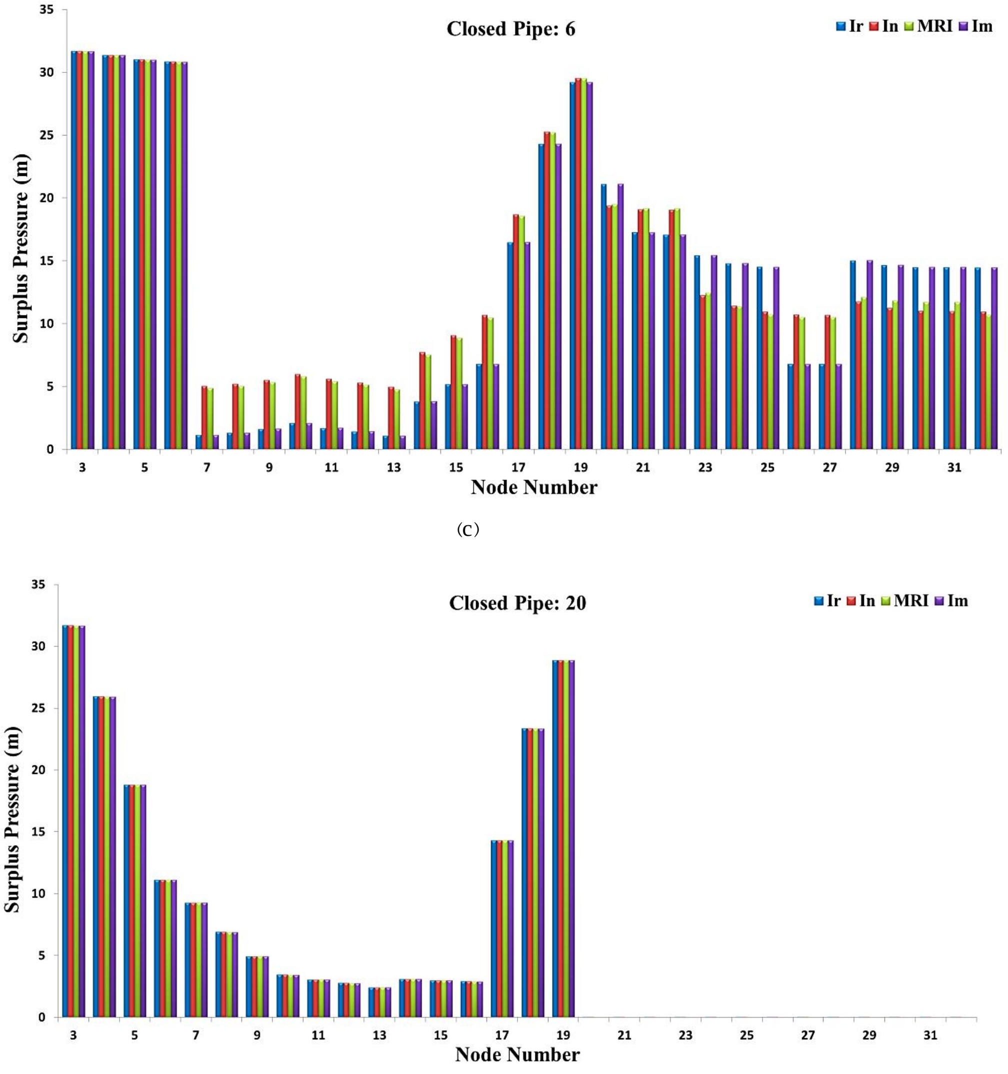
The results of figure 8.(a) and (b) shows that by failure in pipes No. 4 and 5 the surplus pressure in most of the nodes for solutions of maximum I_m criterion is more than solutions with maximum I_r, I_n and MRI. In effect of pipes No. 4 and 5 failures, nodes reactions to pressure changes are similar because these two pipes are along. However, due to failure in pipe No. 6, none of the nodes of the network meet lack of pressure and the figure 8.(c) shows that the solutions with maximum I_n and MRI criteria has more capability to supply pressure in most of the networks. In figure 8.(d) there is no significant difference in represented solutions with reliability criteria values. The nodes with no values in the graph are those that have negative pressures.



(a)



(b)



(c)

(d)

Figure 8. Surplus pressure of Hanoi network nodes for solutions of maximum reliability criteria when pipes No. (a) 4, (b) 5, (c) 6 and (d) 20 are lost due to failure

Example 3:The Real- life network

Real- life WDN is located in Iran and it has 37 pipes, 24 nodes and a reservoir with a 962 m fixed head. The design purpose of this network is municipal water supply of city and improving of the existing condition of the WDN (figure. 9). For this purpose, a series of pipes which have diameters more than 100 mm are used for future conditions (Rasekh, Afshar et al. 2010).For designing this network, polyethylene pipes (PE-80) with Hazen-Williams coefficient of 130 are used. Table 8 presents the nodes and pipes characteristics. Table 9 gives the diameter of these polyethylene pipes as well as cost per unit length. In the design of the network, nodes pressure and velocity constraints are between 14-60 m and 0.2-2 m/s, respectively (Department of Technical Affairs 2013). There are more constraints in this example than the other ones.

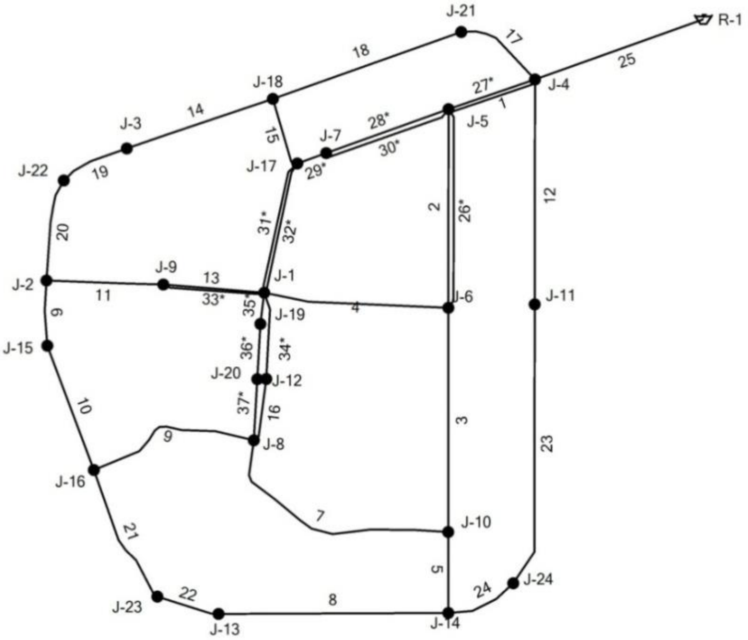


Fig. 9 Layout of Real- life network

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Table 8.Pipes data for real- life WDN

Pipe data	**Pipe**	1	2	3	4	5	6	7	8	9	10	11	12
	Length (m)	73.287	54.62	705	78.58	254.5	205.4	805.5	723.2	556.5	417.2	367.8	707.1
	Pipe	13	14	15	16	17	18	19	20	21	22	23	24
	Length (m)	320.95	485.5	226.4	201.4	292	628.8	225.2	323.0	451.7	200.8	897.3	232.5
	Pipe	25	26*	27*	28*	29*	30*	31*	32*	33*	34*	35*	36*	37*
	Length (m)	9999.87	634.9	299.6	409.0	96.62	423.9	427.0	422.1	318.5	274.6	99.06	174.0	192.3
Node data	**Node**	Reservoir	1	2	3	4	5	6	7	8	9	10	11	12
	Elevation (m)	962	901.5	896.5	895.5	903.5	903.5	902.5	901.5	900	900.5	900.5	903.5	902
	Demand (m^3/h	-301.17	21.78	7.81	20.56	7.56	22.64	30.13	13.25	32.58	18.86	32.8	14.29	0
	Node	13	14	15	16	17	18	19	20	21	22	23	24	
	Elevation (m)	899	905	898	900.5	900.5	899.5	901.5	902	902	898	900.5	904	
	Demand (m^3/h	18.79	9.54	7.45	15.8	11.02	16.31	0	0	0	0	0	0	

* They are existing pipes in the network and are not considered in the total cost of the network.

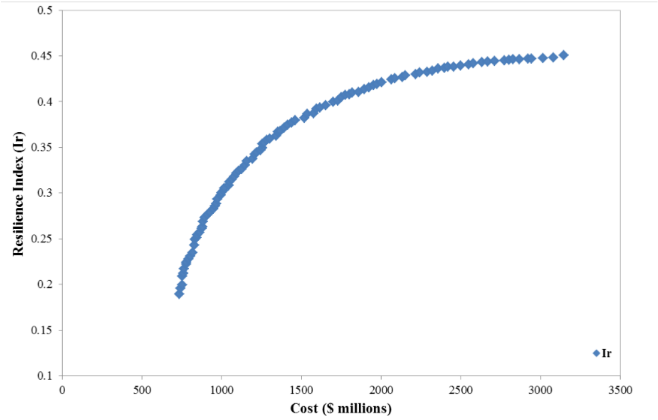
Table 9. Pipe sizes and costs for Real- life network

Pipe	Diameter (mm	Cost (Rial/m)	Pipe	Diameter (mm	Cost (Rial/m)
1	76.6	49560	**6**	191.8	305200
2	93.8	73360	**7**	213.2	375200
3	106.6	94360	**8**	238.8	470400
4	136.4	154000	**9**	268.6	593600
5	170.6	239680	**10**	302.8	753200

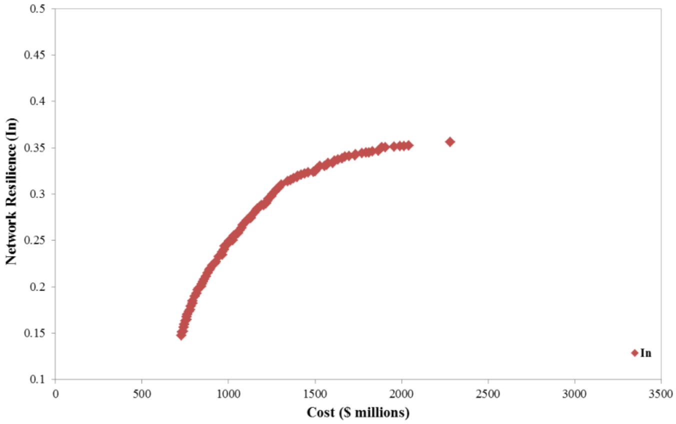
* pipes cost is 28000 Rial/kg based on PE-80 standards with 10 atm pressure.

In the first step, as a result of single-objective optimization using GA in GANetXL, the minimum cost is estimated 7.54×10^8 Rials with 100000 NFEs which shows a cost decrease of 46.14% in comparison to the solution of the consultant company with 14×10^8 Rials (Rasekh, Afshar et al. 2010).

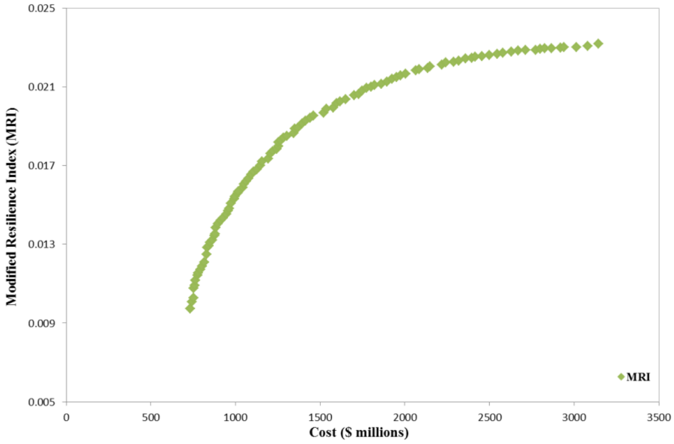
In the second step, the results of figure 10 (a-d) shows that the I_r, MRI, I_m criteria have better performance than I_n criterion for this network in terms of non-dominated solutions. All these three criteria have similar solutions of maximum and minimum cost in the Pareto front. All of the solutions in the Pareto front (figure 10) which obtained by NSGA-II is feasible and satisfied the velocity and pressure constraints.



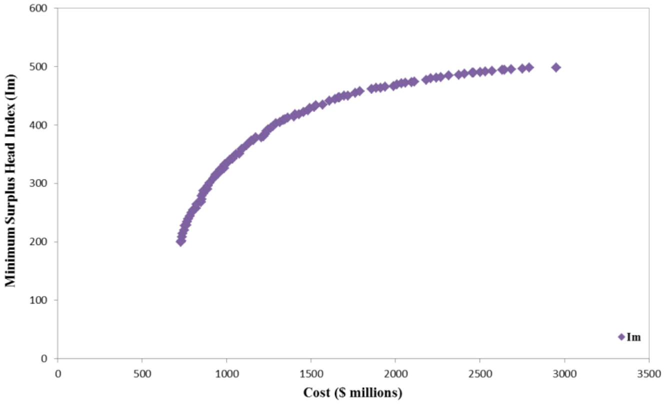
(a)


0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
Network Resilience (In)
0
500
1000
1500
2000
2500
3000
3500
Cost ($ millions)
In

(b)


0.025
0.021
0.017
0.013
0.009
0.005
Modified Resilience Index (MRI)
0
500
1000
1500
2000
2500
3000
3500
Cost ($ millions)
MRI

(c)


600
500
400
300
200
100
0
Minimum Surplus Head Index (Im)
0
500
1000
1500
2000
2500
3000
3500
Cost ($ millions)
Im

(d)

Fig. 10 Pareto front of two-objective function optimization of the Real- life network, (a) Cost-I_r, (b) Cost-I_n, (c) Cost-MRI, (d) Cost-I_m

The results shown in figure 11 demonstrate that in the cost-based optimization, surplus pressure in the nodes number 13 and 23 is less than 1m that explains these nodes are the most critical ones in the network. I_r and MRI criteria have similar and more successful performance compared to I_m in terms of the surplus pressure for all the nodes in the network. I_n has less capability than other criteria to create surplus pressure in the network.

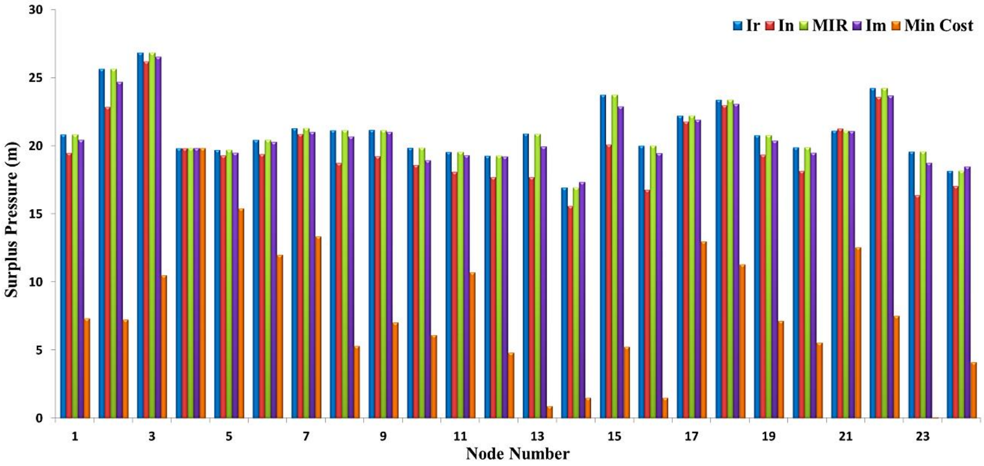


Fig. 11 Surplus pressure of the real- life for solutions of maximum reliability criteria and minimum cost

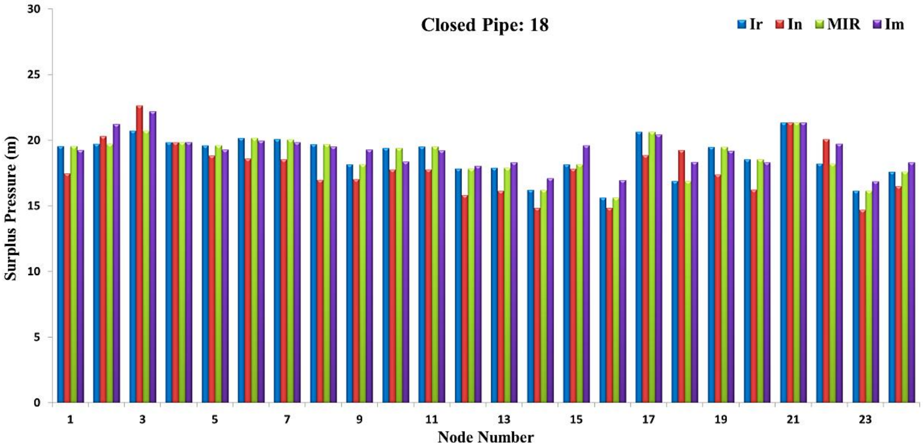
Table 10 presents statistical parameters for new pipes of Real- life Network in result of different runs by GANetXL when optimization approach is cost-based. According to this table pipes No. 18 and 21 were chosen to evaluate the performance of reliability criteria under failure condition because these pipes have less standard deviation and coefficient of variation than other pipes. Moreover, in this network the failure probability should be evaluated in the existing pipes because they have more lifetime in comparison to new pipes. There are different methods accessible to estimate the probability of pipe failure, repair time, and failure return periods. Interested readers should refer to Chapter 18 of Mays (2000). Subsequently, in this study, a random pipe failure has been created using a uniform distribution in the range of [26, 37], that is the pipe numbers for existing pipes (Shafiqul Islam, Sadiq et al. 2013) Finally, the failure of the pipes No. 27 and 34 was analyzed in the network.

Table 10. Statistical parameters for diameters obtained for each new pipe of Real- life network

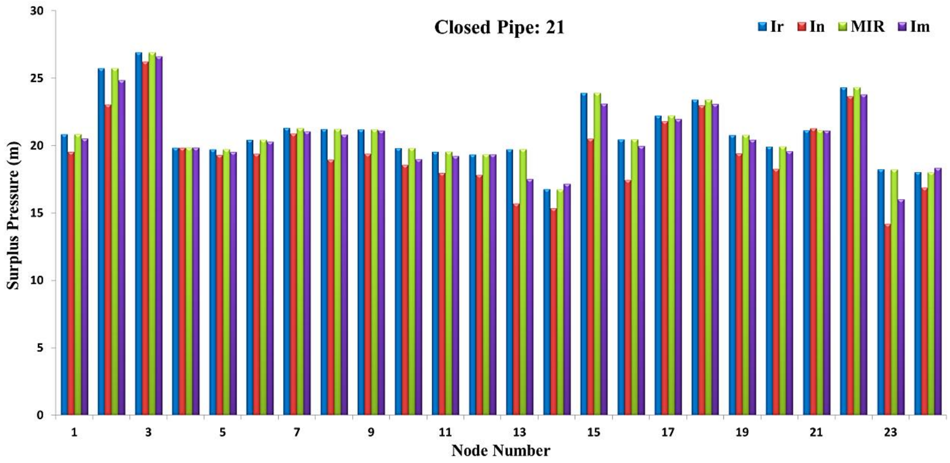
Pipe numb	Maximum (m	Minimum (m	Average (m	ST. DEV	Variance	C.V.
1	213.2	76.6	125.86	21.14	446.99	0.17
2	170.6	76.6	83.30	16.06	257.93	0.19
3	238.8	76.6	140.96	23.97	574.42	0.17
4	302.8	76.6	93.83	25.70	660.65	0.27
5	213.2	76.6	94.09	20.03	401.31	0.21

6	302.8	76.6	112.86	37.87	1433.82	0.34
7	136.4	76.6	97.10	15.48	239.59	0.16
8	268.6	76.6	80.66	21.65	468.56	0.27
9	341.2	76.6	100.11	31.71	1005.72	0.32
10	341.2	76.6	97.53	42.00	1763.92	0.43
11	191.8	76.6	106.91	20.70	428.58	0.19
12	191.8	76.6	97.50	17.47	305.09	0.18
13	403.8	76.6	109.44	39.39	1551.55	0.36
14	213.2	76.6	117.36	29.38	863.00	0.25
15	238.8	76.6	113.80	34.39	1182.59	0.30
16	302.8	76.6	94.20	32.44	1052.59	0.34
17	238.8	76.6	122.40	43.05	1853.03	0.35
18	136.4	76.6	91.02	**11.28**	127.19	**0.12**
19	191.8	76.6	96.02	15.58	242.86	0.16
20	213.2	76.6	94.89	22.87	522.88	0.24
21	136.4	76.6	90.13	**10.38**	107.72	**0.12**
22	302.8	76.6	88.82	32.45	1052.68	0.37
23	268.6	76.6	88.20	31.13	968.85	0.35
24	170.6	76.6	93.74	17.64	311.19	0.19

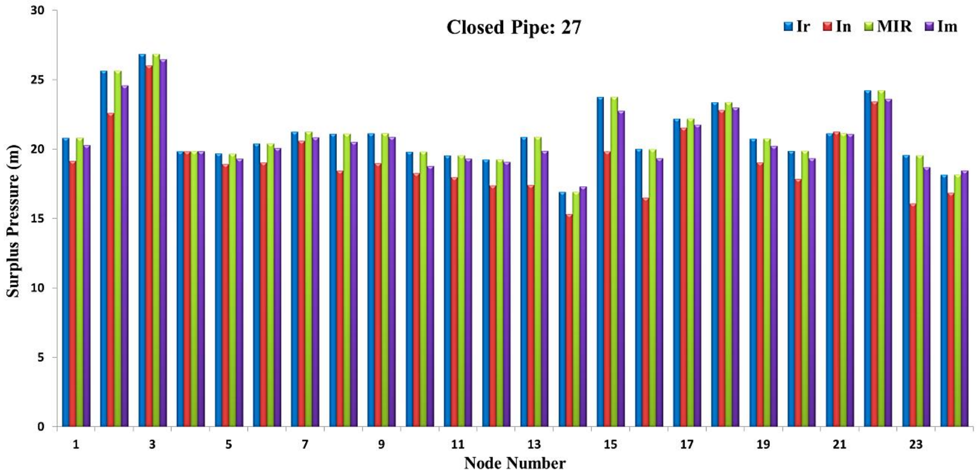
The results of the investigations in figure 12 shows that only the failure in Pipe No. 18 can influence the pressure nodes. Consequently, this pipe is one of the most sensitive pipes in this network. However, reliability performance in the failure conditions is similar to no failure conditions in figure 11. Finally, for this network that includes low diameter in existing pipes, I_n has not a suitable performance because of making the uniformity in pipes connected to a node leads to the decrease of the diameter of new pipes. Thus, the capability of the surplus pressure decreases due to the increase in head-loss in the pipes.



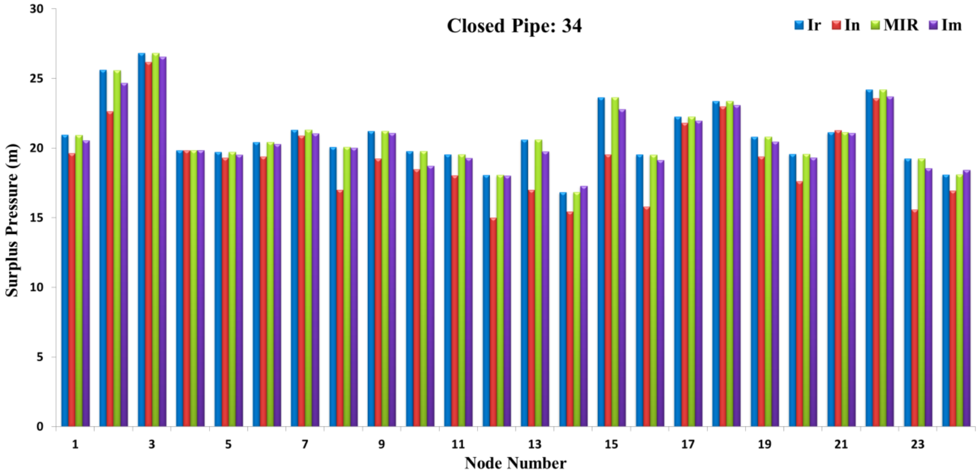
380 (a)


Closed Pipe: 21
Ir
In
MIR
Im
Surplus Pressure (m)
0
5
10
15
20
25
30
1
3
5
7
9
11
13
15
17
19
21
23
Node Number

(b)


Closed Pipe: 27
Ir
In
MIR
Im
Surplus Pressure (m)
0
5
10
15
20
25
30
1
3
5
7
9
11
13
15
17
19
21
23
Node Number

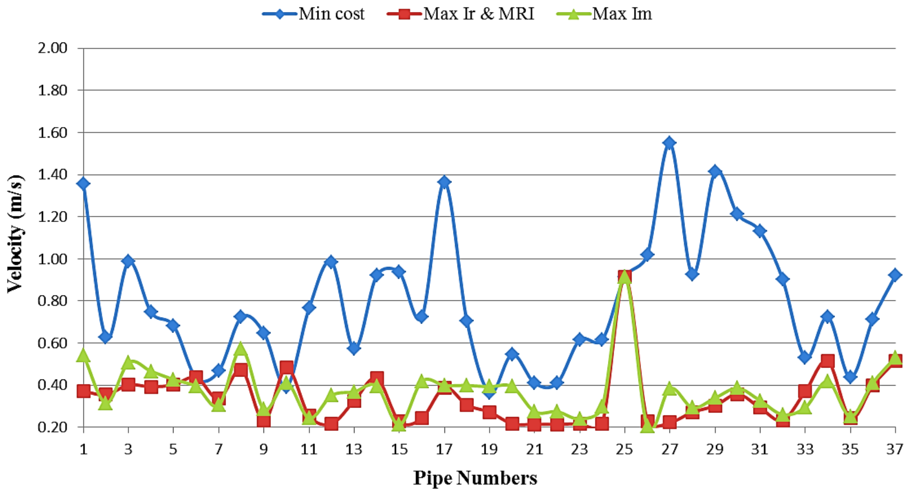
(c)



385

(d)

Fig. 12 Surplus pressure of nodes in Real- life network for solutions of maximum reliability criteria under failure of pipes No. (a) 18, (b) 21, (c) 27 and (d) 34



390

Fig. 13 Velocity variations in pipes for the solutions of minimum cost and maximum I_r, MRI and I_m criteria

Figure 13 shows the velocity variations in the pipes for the solutions with minimum cost and maximum I_r, MRI and I_m criteria obtained using GA and NSGA-II in GANetXL. As it is observed, when the cost is the basis for the design and optimization of Real- life network, velocity variation are so high in the pipes. This can lead to some issues in the network. But

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in the presented solutions with maximum reliability criteria (I_r, MRI and I_m), velocity variations are not only low but almost uniform.

4. Conclusions

In this paper the performance of a few reliability criteria was evaluated when applying to a two benchmark (Two-loop and Hanoi) and one real-life (in Iran) networks. Both the existing pipes and hydraulic constraints were considered in the study in which GANetXL was used as the optimizer. The optimizations were performed taking into account two different objective functions including a cost and reliability.

The results of cost-oriented optimization showed that the solutions proposed by GANetXL for case study networks give solutions that are either less expensive than or as the same as the ones from literature. . In order to investigate the solutions with maximum values of I_r, I_n, MRI and I_m criteria and finding sensitive and important pipes with the most probability of failure in the network, statistical analysis of single-objective optimization was used. The results showed that I_r, MRI and I_m criteria have better performance than I_n under failure conditions, especially in real-life networks that include the existing pipes with small diameter and if a WDN was only optimized based on cost, it would be difficult to overcome losses in pipe failure conditions and pressure supply of nodes.

Competing interests: There is no conflict of interest.

Data Availability: All data and models generated or used during this study are proprietary and confidential in nature.

Author contribution: Conceptualization and data curation were done by A Moghaddam, formal analysis and methodology carried out by A Moghaddam, R Peirovi and H Rezaee, project administration was done by A Faridhosseini and R Peirovi, visualization and writing have prepared by A Naghi Ziaei, A Moghaddam and R Peirovi.

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