

Review Report

Paper Title: Algorithm for Optimization of Branching Gravity-Distribution Networks

Author: Ian Dardani, G. F. Jones

Several algorithms are available in literature for optimal design of branched gravity-fed WDNs. Authors have chosen two out of them, a back tracking (BT) algorithm and a genetic algorithm (GA) without giving any proper justification of their selection. The chosen algorithms have been compared with a new calculus-based algorithm. I would like to suggest authors to provide advantage of proposed CB with those available in literature (Deb 1974, Bhave 1978, Chiplunkar and Khanna 1983, Fujiwara and Dey 1988, Young 1994, Johnson et al. 1996). Bhave's (1978) approach is general and applicable to branched as well as looped networks, gravity as well as pumped source networks, and new as well as existing networks. In case of looped networks, primary pipes forming a branching configuration is identified and designed to carry maximum flows by considering secondary loop-forming links of some minimum size.

Using calculus based approach, Bhave (1978) developed an optimal criterion similar to Eq. (9) of authors and expressed as

{EQUATION FROM BHAVE – not retyped here. Refer to the original review.}

Where ij and jk are supply and distribution links at any node j ; C and h are cost and head loss in any pipe.

Bhave (1978) suggested a univariant method in which nodal heads are assumed initially and corrected iteratively in order to satisfy the optimal criteria at all nodes. Gupta et al. (2003) improved the method of solution adopting Newton-Raphson method in which all correction values are obtained simultaneously for faster convergence of iterative methodology.

The equation (18) of author seems to be similar to Bhave's optimality criterion, if minor losses are ignored. Authors are requested to clearly point out the difference with Bhave's optimality criteria. Also, a systematic procedure or flow chart should be included to apply the proposed methodology to design water networks.

Even though outcome of the paper is general and nothing new in it, the paper can be recommended if the difference between the proposed CB method with Bhave's CB method is clearly indicated and proposed methodology is explained by giving procedure or flowchart.

Additional References

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Fujiwara, O. and Dey, D. (1988). Method of Optimal Design of Branched Network on Flat Terrain. *Journal of Environmental Engineering, ASCE*, 114(6): 1464-1475.

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Johnson, S. L., Gupta, R. and Bhave, P. R. (1996). Discussion of "Design of Branched Water Supply Network on Uneven Terrain" by B. Young. *Journal of Environmental Engineering, ASCE*, 122(5): 448.

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Author's Response:

1. Thank you for your detailed response to our paper.
2. As background, the motivation for cost optimization for the Q-specified condition (we refer to this as a demand-driven design), on which all our designs and studies are based, is twofold:
 - a. To produce the lowest possible cost for a GDWN that satisfies the minimum-head requirement at all nodes subject to the flow rate constraints
 - b. To produce a unique solution to the design problem, which in the absence of cost minimization, produces no unique solution for D . This is noted on pp. 2, 3, and 6 in the paper.
3. The problem considered in this paper is that of selecting a single pipe diameter for each link in a water network to optimize the material cost. This problem has three major categories of methods that are applicable to it: enumeration methods (including both complete enumeration and partial enumeration), nonlinear programming methods, and metaheuristic methods. For each of these categories we have proposed and tested one representative algorithm: backtracking (partial enumeration), the Jones calculus-based algorithm (nonlinear programming), and a genetic algorithm (metaheuristic). While there are many other types of metaheuristic algorithms (simulated annealing, Tabu search, cellular automata, ant colony optimization, and particle swarm optimization), the genetic algorithm is the most representative of these and is also the most commonly used (Zhao et al. 2016).

Note that we have categorized the backtracking algorithm as a partial enumeration method and not a heuristic algorithm. The backtracking algorithm, like heuristic methods, follows a set of deterministic rules to find better solutions, however, those rules are strictly formulated to find cost-optimal solutions and does so without missing the global optimum. In contrast, heuristic algorithms follow rules which achieve some proxy of an optimum solution but do not guarantee the global optimum. One example of a heuristic algorithm comes from Suribabu (2012), whose algorithm uses the uniformity of a solution's flow velocity as a proxy for its cost-optimality. As such, the Suribabu algorithm increments a pipe diameter when its flow velocity is high and decrements the diameter when its flow velocity is low, in an attempt to approach more optimal solutions. It was not necessary to include such a heuristic algorithm for comparison in this paper, since the Backtracking algorithm presented follows a more strictly formulated set of rules which do guarantee a cost optimum.

We note that other methods which can be applied to water network design do not address the problem addressed in this paper. For example, linear programming methods provide split-pipe solutions for each link, while the problem addressed here calls for each link to have a single diameter solution. Multi-objective optimization methods, which typically involve new implementations of metaheuristic methods, are also outside the scope of (single-objective) cost optimization, although it should be noted that cost-optimization algorithms, such as the ones used in this paper, can be used within multi-objective implementations. In addition, decomposition methods, where networks are broken down into smaller sub-networks, can use any of the methods listed above, and are therefore not an exclusive method category. Given the appropriateness of the presented algorithms to the network sizes of gravity water networks, decomposition was not necessary.

We have added a more thorough description to our introduction section that gives clearer context to the selection of these algorithms, highlighting the category of method to which they belong and the key features that make them distinct from one another.

Suribabu, C. R.: Heuristic-based pipe dimensioning model for water distribution networks, J. Pipeline Syst. Eng. Pract., 3(4), 115–124, doi:10.1061/(ASCE)PS.1949-1204.0000104, 2012.

Zhao, W., Beach, T., and Rezgui, Y.: Optimization of Potable Water Distribution and Wastewater Collection Networks: A Systematic Review and Future Research Directions, IEEE Transactions on Systems, Man, and Cybernetics: Systems, 46 (5), 659-681, doi:10.1109/TSMC.2015.2461188, 2016.

4. Equations (15) and (18) are the principal results of our CB modeling, **where (18) includes minor losses** and a two-part pipe cost model, which is more robust compared with the one-part pipe cost model of (15). The greater extent of agreement with pipe cost data shown in Figure 3 affirms this. The one-part pipe cost model is used as a simple case to help with reader understanding of the CB results.
5. At the request of the reviewer, we have the following observations when comparing Equation (18) and, in its more-restrictive form, Equation (15) to the Bhave equation above.
 - a. The fundamental difference of the Bhave approach compared with ours is that Bhave uses an iterative method to solve the design problem and we do not. As such, there are several qualifications leading up to the Bhave equation (as above) in his paper. Among them is that variations in H_j between two iterations is small so that the terms in the cost function may be approximated as constants (Equation (13) in Bhave). This apparently also includes the cost exponent m in the Bhave paper, where normally $m = m(D)$. Upon taking the derivative of Equation (15) in the Bhave paper, m must be assumed constant or else there would result an additive term to each side of Equation (16) in Bhave that is $H_j \ln(H_j) dm/dH_j$ to account for the m -dependence on H_j . This term does not appear. Thus, we reason this is because $\ln(H_j)_{\text{new}} - \ln(H_j)_{\text{old}}$ between two successive iterations is taken to be zero ($(H_j)_{\text{new}}/(H_j)_{\text{old}}=1$). Our method uses our Equations (15) or (18) written at all internal nodes and includes this equation along with the energy equation for each link to solve for the head at each internal node and all link diameters that collectively minimize network cost. The constraint of $h_j \geq h_{\min}$ at each node is, of course, included in the solution. Algorithms to solve a general set of independent, nonlinear algebraic equations using, for example, the Levenberg-Marquardt, Quasi-Newton, Newton-Raphson, or Conjugate Gradient methods are available in most commercial math packages including Matlab and Mathcad (we use the latter with the Quasi-Newton method). In particular, the cost-function coefficient and exponent (b in our paper) are not assumed constant at any node joining two sets of links; see our Equation (18). Nor do we make any assumptions on the orders of magnitude of the terms in our equations to simplify them. In Equation (18) in our paper, the cost-function derivative (Equation (21)), term C' , explicitly accounts for the variation in b . A continuous function (a cubic polynomial; the smallest possible order) is fit over the transition between the small- D and large- D regions of the pipe cost function. Because there is no iterative procedure, there is no flow chart to illustrate our solution. However, based on your comments we have added a few sentences describing the difference between our approach and that of Bhave and a new figure to show the notation ij and jk at a node. Also included is a more-complete description of our method of solution. We believe these additions have added to the level of quality of our paper. Thank you.
 - b. Because of the lack of restrictions of Equations (15) and (18) in our paper, the CB algorithm can be applied to any pipe network including serial, branch, and loop. For serial networks, in particular, where the flow rate across a node is constant, one can clearly see from inspection of Equation (15) the optimal solution of a constant hydraulic gradient (S_{ij}) or $D_{ij} = D_{jk} = \text{constant}$ for any two links provided $h_j \geq h_{\min}$.
 - c. The development of Equation (16) in Bhave is not explicitly given, as the steps between his Equations (15) and (16) are not shown in his paper. Conversely, all steps in the development of our Equation (15) appear in our paper, so readers are unambiguously informed of the origin of this equation and its variant, Equation (18).

Algorithms for Optimization of Branching Gravity-Driven Water Networks

Ian Dardani¹, Gerard F. Jones¹

¹Villanova University, College of Engineering, Villanova, 19085, United States

5 *Correspondence to:* Ian Dardani (ian.dardani@villanova.edu)

Abstract. The design of a water network requires the selection of pipe diameters that satisfy pressure and flow requirements while optimizing for cost. This work focuses on the design of moderate-scale branching Gravity-Driven Water Networks (GDWNs), in contrast to large urban-scale looping networks, where budgets are highly constrained and where Polyvinyl chloride (PVC) pipe is typically used. To help designers of GDWNs select an appropriate design approach for a given network problem, three cost-minimization algorithms are presented and results compared with five GDWN test cases. Two algorithms, a backtracking algorithm and a genetic algorithm, use a set of discrete pipe diameters, while a new calculus-based algorithm produces a continuous-diameter solution, which is mapped onto a discrete-diameter solution. The backtracking algorithm produced the overall lowest-cost solutions with relative efficiency for the test cases, while the calculus-based algorithm produced slightly higher-cost results but with greater scalability to networks with more links. Furthermore, the new calculus-based algorithm's continuous-diameter and mapped solutions provided lower and upper bounds, respectively, on the discrete-diameter global optimum cost, where the mapped solutions were typically within one diameter size of the global optimum. Overall, the genetic algorithm as implemented did not produce results which deemed it compelling over deterministic methods for GDWN design. However, for more complex networks and problem formulations, a genetic algorithm may be more advantageous, particularly if it incorporates improvements reported in the literature. The results of this study highlight the advantages and weaknesses of each GDWN design method including closeness to the global optimum, the ability to prune the solution space of infeasible and sub-optimal candidates without missing the global optimum, and calculation time. We also extend an existing closed-form model of Jones (210) to include minor losses, a more-comprehensive two-part cost model, which realistically applies to pipe sizes that span a broad range typical of GDWNs of interest in this work, and for smooth and commercial steel roughness values.

25 1 Introduction

A gravity-driven water network (GDWN) is commonly constructed to deliver potable water to a community in a developing region. These systems draw water from a source at a high elevation, such as a natural spring or a stream, and deliver it through a branching pipe network to household taps or public tapstands (Fig. 1). In principal, loops and

loop/branching constructs may be added to networks for greater reliability, but material cost considerations often restrict attention to just branch networks in GDWNs. The methodologies presented in this paper, however, may be extended to all networks, including those with loops. When feasible, gravity water networks are very attractive compared with pumped networks because of their simplicity and lower capital, operational, and maintenance costs. In addition, in most locations where GDWN are considered, there may be little or no access to reliable grid-based electrical power for pumps.

Water networks are modelled as a collection of nodes, each representing a point of water demand or supply, which are connected with links representing pipes. Typically, the layout of the site is known, including water source and demand locations and elevations of all other nodes. For the present work, design flow rates are determined from community survey data, which are extrapolated for future population growth. Networks in this category are referred to as “demand-driven” designs. Bhave (1978) refers to these as “Q-specified” designs. Thus, to design a network of this type, pipe diameters for each link must be chosen such that acceptable but arbitrary minimum pressure heads are maintained at each node given a design flow rate at the node. Furthermore, application of the energy equation to this network demonstrates that the design problem is non-unique; i.e., choosing different pressure heads at the nodes will result in a different pipe diameter solution for the network, and thus different networks costs.

In practice, gravity-driven water networks are commonly designed by a marching method, where diameters for each link of the network are chosen sequentially. After selecting a reasonable diameter for each link, the designer calculates the static pressure head at the link outlet, and proceeds to the next link if this result is acceptable. In this way, the designer marches through the network until all pipe diameters have been selected. This method produces a feasible solution, but not a cost-optimized one. As noted by Bhave (2003), cost savings of 20-30% can result from the use of optimization techniques. In developing regions, the cost of a water network can be prohibitive, adding to the importance of optimizing network design.

Within the provided framework, the global optimum can be found through an exhaustive search of the solution space, known as complete enumeration, although this is infeasible when considering networks with many links and diameter choices (Kadu et al. 2008; González-Cebollada 2011). To reduce the computational time required by enumeration, authors have proposed various partial enumeration methods which prune the search space (Kadu et al. 2008), although some of these techniques may remove the global optimum (Simpson et al. 1994). The most common types of algorithms that have been applied to optimize water network design include traditional deterministic methods, heuristic methods, metaheuristic methods, multi-objective methods, and decomposition methods (Zhao et al. 2016).

Deterministic methods include linear programming (LP), dynamic programming, and nonlinear programming (NLP), and typically involve rigorous mathematical approaches (Zhao et al. 2016). A brief overview and comparison of these algorithms is given in Kansal, et al. (1996), who use a single-part cost correlation for metric pipe diameters between 100 mm and 350 mm. Linear programming techniques have relatively low computational complexity and allow each link to be composed of two diameters, called a split-pipe solution, although these may not always be practical to implement (Kessler and Shamir 1989, Swamee and Sharma 2000, Samani and Mottaghi 2006). LP can also get stuck in a local optimum (Zhao et al.

2016), although combining LP with metaheuristic techniques can help with the problem's non-smoothness properties (Krapivka and Ostfeld 2009). Dynamic programming has been used by Yang et al. (1975) and Martin (1980) to optimize networks in stages. This approach begins at the discharge nodes, proceeding to select feasible diameters and joints for upstream stages and storing these partial candidates in memory until the source node is reached. At this point, the algorithm reviews the
5 feasible segment design options and selects a combination of stage solutions producing the lowest cost overall solution. This method, however, requires the designer to allow a relatively narrow range for the design pressure of each node, or otherwise store a large set of feasible candidate solutions in memory and also allow adjoining branches to arrive at different heads at the same node.

Nonlinear programming, a calculus-based method, deals with each link's diameter as a continuous variable. Using
10 Lagrange multipliers and a one-part, pipe-cost model with minor-lossless flow, Swamee and Sharma (2000) developed systems of equations for both continuous and discrete pipe diameters for branch networks, assuming constant friction factor. When solved, the solution gives diameter values that minimize distribution main cost, not network cost. In carrying out the solution, iteration is required to update the value of the friction factor. For the discrete diameter case, large computational times were noted by Swamee and Sharma because of the stiffness of the mathematical system. Cases where one or more nodal pressure
15 heads are not acceptable need to be treated manually by the designer in various ways as discussed by the authors.

For branching networks, Jones (2011) showed that by restricting the focus to smooth-turbulent, minor-lossless flow, and the use of a one-part, pipe-cost model, a simple nonlinear algebraic equation for each internal node in the distribution main could be developed. The equation has been extended in the present work to include minor losses and rough pipe. When solved simultaneously with the energy equation for each link, a unique solution for all link diameters and nodal pressure head values
20 are obtained that produces minimum network cost, as opposed to the distribution main cost as in Swamee and Sharma (2000). The method of Jones (2011) also applies to serial and loop networks because of its generality.

Heuristic methods follow specific rules to incrementally build better solutions, although the rules are not strictly formulated to trend towards local or global optima. An approach by Monbaliu et al. (1990) sets all network pipes to their minimum size, where the pipe that has a maximum head loss gradient is incremented to its next-highest size until all nodal
25 head requirements are satisfied. Similarly, an algorithm by Keedwell and Khu (2006) selects an initial solution and iteratively responds to nodal head deficits and surpluses by incrementing or decrementing pipe sizes accordingly until a feasible solution is found. Suribabu (2012) proposed a heuristic that identifies pipes to increment or decrement in size based on flow velocity and alternative metrics such as proximity to the source node, achieving acceptable cost results with computational efficiency. While these algorithms are typically computationally efficient, they do not guarantee a global optimum.

30 Metaheuristic optimization methods allow for a set of solutions to evolve through random processes that are guided with an objective function which rewards low network costs and penalizes hydraulic insufficiencies. Examples include evolutionary algorithms, which are most commonly genetic algorithms (Krapivka and Ostfeld 2009, Simpson et al. 1994, Kadu et al. 2008, Prasad and Park 2004), simulated annealing (Vasan and Simonovic 2010; Tospornsampan et al. 2007), ant colony

optimization (Maier et al. 2003), and differential evolution (Vasan and Simonovic 2010). As reviewed by Nicklow et al. (2010), evolutionary algorithms are an emerging popular alternative to the deterministic methods, and they offer the opportunity to accommodate unique constraints and multiple design objectives. The main challenges for evolutionary algorithms are the difficulty of incorporating constraints into objective functions, the optimum selection of parameters, and a relatively large amount of computational effort. In addition to optimizing for cost, multi-objective methods, often based on evolutionary algorithms, allow the designer to choose from a Pareto-optimal front of objectives, such as cost and reliability (Prasad and Park 2004). In addition to water network design, metaheuristic algorithms have been used for a range of problems in water resources engineering, such as rainfall and runoff modelling (Taormina et al. 2015).

Decomposition methods involve the partitioning of networks into smaller sub-networks which are each optimized using one of many types of techniques and then combined into an overall solution. In some cases, the loops in the sub-networks are removed, producing branching trees which are then optimized individually. Techniques used to optimize the sub-networks can involve multiple methods, including linear programming (Saldarriaga 2013) and differential evolution (Zheng et al. 2013), with a later stage optimizing the network as a whole using the sub-network solutions as inputs. Note that another distinct use of the term ‘decomposition’ refers to the approach of iteratively solving “inner” and “outer” mathematical problem formulations, and has been used in the literature by Krapivka and Ostfeld (2009) who traces its use in this context back to Alperovits and Shamir (1977).

In the present study, we present three algorithms, each from one of three major categories of methods applied to cost optimization of water distribution networks, and compare their performance on five cases adapted from real GDWNs. These algorithms include (1) the calculus-based (CB) optimization model of Jones (2011), an NLP method, (2) backtracking (BT), a partial enumeration method, and (3) a genetic algorithm (GA), a metaheuristic method. Major distinguishing features of these algorithms include their working use of continuous diameters (CB) versus discrete diameters (BT and GA), their deterministic nature (CB and BT) versus a stochastic nature (GA), and their relative scalability as better (CB, GA) and worse (BT) for larger networks. In terms of their ability to find a global optimum solution for the problem formulation, CB finds a global optimum for continuous diameters but cannot guarantee a discrete diameter global optimum in its mapped solution, BT can guarantee a discrete global optimum, and GA cannot guarantee an optimum. For a direct comparison of techniques, the pipe costs used for all algorithms are found by interpolating a two-part cost formula based on a curve-fit of real cost data for available diameter values. The three algorithms are tested against networks adapted from field data on five actual GDWNs installed in Panama, Nicaragua, and the Philippines.

Within the broader context of water network problem formulations, this paper is concerned with finding cost-optimal single-diameter solutions to branching water distribution networks with steady-state demand flows and pre-specified pipe locations.. By implication of being gravity-driven, the problem does not involve the use of pumping stations. This problem formulation is directly applicable to typical gravity-driven water networks, and is also useful for multi-objective algorithms,

the consideration of sub-networks in a decomposition technique, pumped networks, and looped system optimization, which can involve reformulating the problem into a branching configuration.

The results of this study highlight the advantages and weaknesses of each GDWN design method including closeness to the global optimum, the ability to prune the solution space of infeasible and sub-optimal candidates without missing the global optimum, and also computational time. We present two pre-processors with which discrete-diameter search methods can use to reduce the search space without pruning the global optimum. To the authors' knowledge, is the first implementation of Pre-Processor 1 in enumeration methods and the first implementation of Pre-Processor 2 in any method. We also extend the Jones closed-form model to include minor losses, a more-comprehensive two-part cost model, which realistically applies to pipe sizes that span a broad range typical of GDWNs of interest in this work, and for smooth and commercial steel roughness values.

2 Problem Formulation

Branching networks are considered (Fig. 1), where all branches connect a distribution main node with a delivery node, shown as tapstands or houses. For each link in a network of N_L links, pipe length (L) and the net elevation change (Δz) are considered fixed. Steady-state flow rates (Q) are prescribed for each link based on the demand flow data at delivery nodes. As noted above, demand flows are determined by community surveys and extrapolated in time to quantitatively account for population growth. Minor losses are accounted for through a minor loss coefficient K or a dimensionless equivalent pipe length, (L_e/D , or in symbol form, L_{ebyD}), where L_e is the pipe length of diameter D whose frictional loss results in the corresponding minor loss. An optimal solution is obtained by selecting pipe diameters (D) from a set of commercially available diameters such that the network's material cost is minimized. With N_D choices of diameters for N_L links, the problem has $N_D^{N_L}$ candidate solutions.

For all nodes, static pressure, h , is greater than or equal to a chosen minimum, h_{min} . The value for h_{min} is selected to eliminate possible leakage of contaminated ground water into the network should the operating conditions change in an unanticipated way. The change in static pressure head, Δh , across each link is calculated with the energy equation for pipe flow,

$$\Delta h = -\Delta z + \left(\alpha + K + f \left(\frac{L}{D} + L_{ebyD} \right) \right) \frac{8Q^2}{\pi^2 g D^4} \quad (1)$$

where for each link, α is the kinetic energy correction factor and f is the Darcy friction factor, calculated with the Colebrook-White equation (Colebrook and White 1937) or Churchill correlation (Churchill 1977), and g is acceleration of gravity. The kinetic energy correction factor, α , is considered only in the first link, where acceleration from a zero-velocity source is sometimes non-negligible for the smallest of GDWNs that have been encountered. Thus,

$$\alpha = \begin{cases} 2 & \text{Re} \leq 2300 \\ 1.05 & \text{Re} > 2300 \end{cases}$$

where Re is the Reynolds number for pipe flow, $4Q/\pi \nu D$, and ν is the kinematic viscosity of water. The possibility of laminar flow ($\text{Re} \leq 2300$) is permitted since branches from the smallest GDWN observed in practice have been in this regime.

The pressure upper bound is not incorporated into the optimization process. Worst-case pressure conditions occur under hydrostatic conditions, which are directly related to the maximum elevation change in the network and where no flow occurs. Therefore, before the optimization process is undertaken, the selections of appropriate pressure ratings for the pipe and, if needed, break-pressure tanks are left to the correct judgment of the designer under no-flow conditions. In addition, precautions against water hammer are left to the designer.

3 New Calculus-Based Algorithm

In this section we develop a new calculus-based algorithm for pipe diameters that minimize overall pipe cost for the network. First appearing in the text by Jones (2011), this algorithm is solved simultaneously with the energy equation for each link to produce unique solutions for D and nodal pressure head values that minimize network pipe cost, as opposed to only the distribution main cost as in Swamee and Sharma (2000). The method also applies to serial and loop networks.

First consider the physical basis for the existence of a unique set of pipe diameters and static pressures for the demand-driven design problem with cost minimization included. Several works reviewed in the previous section have considered optimization of GDWN and combined pumped and gravity-driven networks. We assume continuous pipe diameters in this section; values that result from the solution of the energy equation. Mapping between continuous diameters and the discrete nominal sizes, required to complete the design, will not be fully addressed in the present work. However, we will discuss two methods we have used for mapping the continuous D solutions onto the discrete pipe diameter set.

Consider the three-pipe network shown in Fig. 2. Pipes 1-2, 2-3, and 2-4 meet where head h_2 is unknown. Each pipe has prescribed volume flow rate and length and unknown diameter D as shown. The change in elevation between the top and bottom of each pipe is Δz and Δh is the change in static pressure head. There is a prescribed head at each outlet for pipes 2-3 and 2-4.

To facilitate insight, we at first assume turbulent flow, which can be verified post-calculation if necessary, *in smooth pipe* and that minor losses are negligible. Two sources for the friction factor for smooth-turbulent flow are considered, namely the classical Blasius equation (reported in Streeter et al. 1998), $f = 0.316 \text{Re}^{-1/4}$, and the Swamee-Jain correlation (Swamee and Jain 1976), $f = 0.175 \text{Re}^{-0.1923}$ (though not explicitly appearing in this reference, f from the Swamee-Jain correlation is obtained by writing it for smooth pipe and comparing this with the energy equation, where f is assumed to be in the form $a\text{Re}^n$). The Blasius equation has higher accuracy (2% for low Re and 3% for high Re) in the range $10^4 < \text{Re} < 10^5$, over which most of the GDWNs in this work operate, compared with the Swamee-Jain correlation of +8% / -3%, thus prompting the Blasius equation to be chosen for this work. A combination of the Blasius equation with the energy equation gives explicit

formulas for D for the three links in Fig. 2. For simplicity, and to reduce the number of free parameters, the conditions for pipes 2-3 and 2-4 are assumed to be identical without loss of generality. We obtain

$$D_{12} = 0.741 \left(\frac{\Delta z_{12} + \Delta h_{12}}{L_1} \right)^{-4/19} \left(\frac{Q_{12} v^{1/7}}{g^{4/7}} \right)^{7/19} \quad (2)$$

$$D_{23} = D_{24} = 0.741 \left(\frac{\Delta z_{23} + \Delta h_{23}}{L_2} \right)^{-4/19} \left(\frac{Q_{23} v^{1/7}}{g^{4/7}} \right)^{7/19}$$

With our assumptions and inspection of Fig. 2, $\Delta h_{12} = -h_2$ and $\Delta h_{23} = \Delta h_{24} = h_2 - h_3 = h_2 - h_4$, obtain

$$D_{12} = 0.741 \left(\frac{\Delta z_{12} - h_2}{L_1} \right)^{-4/19} \left(\frac{Q_{12} v^{1/7}}{g^{4/7}} \right)^{7/19} \quad (3)$$

$$D_{23} = D_{24} = 0.741 \left(\frac{\Delta z_{23} - h_3 + h_2}{L_2} \right)^{-4/19} \left(\frac{Q_{23} v^{1/7}}{g^{4/7}} \right)^{7/19}$$

The pipe cost model can be assumed to follow a power-law relationship (Swamee and Sharma 2008)

$$C' = a \left(\frac{D}{D_u} \right)^b \quad (4)$$

5 where a is a constant coefficient, b is a constant exponent, and D_u an assumed unit diameter. A more robust, two-part model, valid for a greater range of pipe sizes than that of Swamee and Sharma (2008), will be used below. The use of pipe material cost as the objective function was assumed because of relevance. In most GDWNs of interest in this work, installation labor comes from the local community and has no well-defined associated cost. The material cost for the network is of prime importance since it normally comes from funds raised by nongovernmental organizations or grants, where there is seldom a required repayment but are always in short supply. For a more-general case, the economics of a GDWN may be more encompassing and include materials, labor, operation and maintenance, depreciation, taxes, and salvage, among others. The time value of money may also need to be considered, which includes interest rates and estimation of the network lifetime.

10 With Eq. (4) the general expression for the total cost for the pipe material, C_T , is obtained by summing over all links ij ,

$$C_T = a \sum_{ij} L_{ij} \left(\frac{D_{ij}}{D_u} \right)^b \quad (5)$$

15 which, for the present problem, becomes

$$C_T = a \left[L_{12} \left(\frac{D_{12}}{D_u} \right)^b + L_{23} \left(\frac{D_{23}}{D_u} \right)^b + L_{24} \left(\frac{D_{24}}{D_u} \right)^b \right] \quad (6)$$

$$= a \left[L_{12} \left(\frac{D_{12}}{D_u} \right)^b + 2L_{23} \left(\frac{D_{23}}{D_u} \right)^b \right]$$

A close inspection of Eq. (3) in combination with Eq. (6) will reveal the origin of the existence of an optimal h_2 for the design of the network in Fig. 2. Because of its arbitrariness, we are free to vary the value of h_2 . As h_2 increases, say from a small value like 1 m, the pressure difference between the junction and the bottom of pipe 2-3 (and 2-4) increases. Since the

volume flow rates in each pipe are fixed, an increase in pressure drop across pipe 2-3 (and 2-4) requires a reduction in D_{23} (and D_{24}). This is evident from our inspection of the second of Eq. (3), where we see that D_{23} and D_{24} are both proportional to $(\Delta z_{12} - h_3 + h_2)^{-4/19}$; that is $(\Delta z_{12} - h_3 + h_2)^{-4/19}$ decreases as h_2 increases.

Because the top of pipe 1-2 is at atmospheric pressure, an increase in h_2 will decrease the pressure drop between the top of pipe 1-2 and the junction. Thus, compared with pipes 2-3 and 2-4, the opposite effect occurs in pipe 1-2; D_{12} increases with increasing h_2 . For insight on how the energy equation supports this explanation, note that the first of Eqs. (3) requires that $D_{12} \approx (\Delta z_{12} - h_2)^{-4/19}$ increases as h_2 increases.

From this discussion it is clear that for an increasing h_2 there is a competition between the decrease of D_{23} (and D_{24}) and an increase in D_{12} . Once the effect of D on pipe cost is included through Eq. (6), as h_2 increases we see that the cost for pipes 2-3 and 2-4 decrease, and the cost for pipe 1-2 increases. A consequence of this competition is the existence of an optimum, in this case an optimal h_2 , which produces the smallest possible cost.

The mathematical basis for a unique solution for h_2 with cost minimization is now presented. In addition to the fixed pipe lengths, the total cost depends on the diameters for all of the pipes in the network. For the case of Fig. 2, where we now allow pipe 2-3 and pipe 2-4 to be different, get

$$C_T = C_T(D_{12}(h_2), D_{23}(h_2), D_{24}(h_2)) \quad (7)$$

Using the chain rule from the calculus, the total differential of Eq. (7) is

$$dC_T = \frac{\partial C_T}{\partial D_{12}} \frac{\partial D_{12}}{\partial h_2} dh_2 + \frac{\partial C_T}{\partial D_{23}} \frac{\partial D_{23}}{\partial h_2} dh_2 + \frac{\partial C_T}{\partial D_{24}} \frac{\partial D_{24}}{\partial h_2} dh_2 \quad (8)$$

The minimum value of C_T is found once $dC_T = 0$ (and once it is verified that the second derivative of C_T is positive thus indicating that C_T is indeed a minimum). Requiring this, obtain

$$0 = \frac{\partial C_T}{\partial D_{12}} \frac{\partial D_{12}}{\partial h_2} + \frac{\partial C_T}{\partial D_{23}} \frac{\partial D_{23}}{\partial h_2} + \frac{\partial C_T}{\partial D_{24}} \frac{\partial D_{24}}{\partial h_2} \quad (9)$$

The cost C_T is from Eq. (5), so the derivatives like $\partial C_T / \partial D_{12}$ in Eq. (9) are written in general as

$$\frac{\partial C_T}{\partial D_{ij}} = a b \frac{D_{ij}^{b-1}}{D_u^b} L_{ij} \quad (10)$$

for any link ij .

The derivatives like $\partial D_{12} / \partial h_2$ in Eq. (9) are obtained by taking the partial derivative of the pipe diameter with respect to the relevant pressure head in the appropriate energy equation. For the full energy equation, where D appears in a nonlinear

way in more than one location, this would be done using numerical methods. However, if we restrict our interest to minor-lossless, smooth-turbulent flow as noted above, we can use the energy equations like Eq. (3). Obtain for pipe 1-2

$$\frac{\partial D_{12}}{\partial h_2} = 0.156 \left(\frac{\Delta z_{12} - h_2}{L_{12}} \right)^{-23/19} \left(\frac{v^{1/7} Q_{12}}{g^{4/7} L_{12}^{19/7}} \right)^{7/19} \quad (11)$$

For pipe 2-3, we get

$$\frac{\partial D_{23}}{\partial h_2} = -0.156 \left(\frac{\Delta z_{23} + h_2 - h_3}{L_{23}} \right)^{-23/19} \left(\frac{v^{1/7} Q_{23}}{g^{4/7} L_{23}^{19/7}} \right)^{7/19} \quad (12)$$

and for pipe 2-4,

$$\frac{\partial D_{24}}{\partial h_2} = -0.156 \left(\frac{\Delta z_{24} + h_2 - h_4}{L_{24}} \right)^{\frac{23}{19}} \left(\frac{v^{1/7} Q_{24}}{g^{4/7} L_{24}^{19/7}} \right)^{\frac{7}{19}} \quad (13)$$

5 Equations (10)–(13) are combined with Eq. (9) to produce a single algebraic equation that depends on h_2 , as well as D_{12} , D_{23} , and D_{24} . Introducing D_{12} , D_{23} , and D_{24} from Eqs (3) into this single algebraic equation, we get

$$\begin{aligned} 0 = & Q_{12}^{7b/19} \left(\frac{\Delta z_{12} - h_2}{L_{12}} \right)^{-(1+4b/19)} - Q_{23}^{7b/19} \left(\frac{\Delta z_{23} + h_2 - h_3}{L_{23}} \right)^{-(1+4b/19)} \\ & - Q_{24}^{7b/19} \left(\frac{\Delta z_{24} + h_2 - h_4}{L_{24}} \right)^{-(1+4b/19)} \end{aligned} \quad (14)$$

The general form of Eq. (14), written at any internal node is

$$0 = \sum_{ij,in} Q_{ij}^{7b/19} S_{ij}^{-(1+4b/19)} - \sum_{ij,out} Q_{ij}^{7b/19} S_{ij}^{-(1+4b/19)} \quad (15)$$

where the hydraulic gradient, S_{ij} , is

$$S_{ij} = \frac{\Delta z_{ij} + \Delta h_{ij}}{L_{ij}} \quad (16)$$

In Eq. (15) the indices ij,in and ij,out on the summations refer to inflows and outflows at the node (e.g., in Fig. 2, ij,in = 12 and ij,out = 23 and 24). Equation (15), the new CB algorithm proposed in this work, is written for each internal node in the network and solved simultaneously with the energy equation for each link to obtain unique and optimal values of D_{ij} for all links and h_j for all internal nodes. It is understood that the nodal pressure heads determined from the solution of this system must be greater than or equal to the h_{min} prescribed for the network. For nodes that do not satisfy this condition, the pressure head is set equal to h_{min} , as part of the CB algorithm. Thus $h_j \geq h_{min}$.

15 Minor losses using the equivalent-length method can be included in the above developments by artificially extending the length of the link by L_e in which minor loss occurs, thus contributing a non-zero $L_{e\text{byd}}$ term in Eq. (1). We also extend the cost model of Eq. (5) from Swamee and Sharma (2008) to encompass two different ranges of pipe diameters having two

different coefficients a and exponents b . The link between the two ranges starts at discrete pipe size D_{co} , at and below which the cost model for the small (subscript s) pipe sizes applies, and discrete pipe size D_{co+1} , at and above which the cost model for the large (subscript l) pipe sizes applies. The cutoff diameter, D_{co} is chosen by the designer based on inspection of cost vs. diameter data. Thus,

$$C_{ij} = L_{ij} \begin{cases} a_s \left(\frac{D_{ij}}{D_u}\right)^{b_s}, & D_{ij} \leq D_{co} \\ c_1 + c_2 \frac{D_{ij}}{D_u} + c_3 \left(\frac{D_{ij}}{D_u}\right)^2 + c_4 \left(\frac{D_{ij}}{D_u}\right)^3, & D_{co} < D_{ij} < D_{co+1} \\ a_l \left(\frac{D_{ij}}{D_u}\right)^{b_l}, & D_{ij} \geq D_{co+1} \end{cases} \quad (17)$$

- 5 In Eq. (17), a_s and a_l are the coefficients for the small and large pipe size regions, respectively, and b_s and b_l are the exponents for the small and large pipe size regions, respectively. A cubic spline is fit between pipe sizes D_{co} and D_{co+1} to complete the transition between small and large pipe sizes. The coefficients of this polynomial are c_1 , c_2 , c_3 , and c_4 as seen in Eq. (17). These coefficients are evaluated by matching the cubic polynomial and pipe data at D_{co} and D_{co+1} and the first derivative of the polynomial with respect to D_{ij}/D_u to $a_s b_s \left(\frac{D_{co}}{D_u}\right)^{b_s-1}$ at $D_{ij} = D_{co}$ and to $a_l b_l \left(\frac{D_{co+1}}{D_u}\right)^{b_l-1}$ at $D_{ij} = D_{co+1}$. An
- 10 example of data for Polyvinyl chloride (PVC) pipe and the curvefit is shown in Fig. 3. The results of the curvefit are: $D_{co} = 2.067$ in., $D_{co+1} = 2.469$ in., $a_s = \$1.349 \text{ m}^{-1}$, $b_s = 1.157$, $a_l = \$1.381 \text{ m}^{-1}$, $b_l = 1.344$, $c_1 = \$237.516 \text{ m}^{-1}$, $c_2 = -\$316.125 \text{ m}^{-1}$, $c_3 = \$140.450 \text{ m}^{-1}$, $c_4 = -\$20.499 \text{ m}^{-1}$. It is clear from inspection of Fig. 3 that a one-part cost model would not have produced an acceptable curve-fit to pipe-cost data.

With the inclusion of the two-part cost model and minor loss term, Eq. (15) becomes

$$0 = \sum_{ij,in} \frac{C'_{ij} A_{ij}^{\frac{4}{19}} (1+\epsilon_{ij})^{\frac{4}{19}} S_{ij}^{\frac{23}{19}} \left(\frac{Q_{ij}^7 v}{g^4 D_u^{19}}\right)^{\frac{1}{19}}}{1-B A_{ij}^{\frac{4}{19}} \epsilon_{ij}' (1+\epsilon_{ij})^{-\frac{15}{19}} S_{ij}^{-\frac{4}{19}} \left(\frac{Q_{ij}^7 v}{g^4 D_u^{19}}\right)^{\frac{1}{19}}} - \sum_{ij,out} \frac{C'_{ij} A_{ij}^{\frac{4}{19}} (1+\epsilon_{ij})^{\frac{4}{19}} S_{ij}^{\frac{23}{19}} \left(\frac{Q_{ij}^7 v}{g^4 D_u^{19}}\right)^{1/19}}{1-B A_{ij}^{\frac{4}{19}} \epsilon_{ij}' (1+\epsilon_{ij})^{-\frac{15}{19}} S_{ij}^{-\frac{4}{19}} \left(\frac{Q_{ij}^7 v}{g^4 D_u^{19}}\right)^{1/19}} \quad (18)$$

- 15 where $B = 0.1989$ and

$$\epsilon_{ij} = \sum_k \left(\frac{L_e}{D}\right)_{k,ij} \frac{D_{ij}}{L_{ij}} \quad (19)$$

$$\epsilon_{ij}' = \sum_k \left(\frac{L_e}{D}\right)_{k,ij} \frac{D_u}{L_{ij}} \quad (20)$$

$$A_{ij} = \begin{cases} 0.318, & \text{smooth pipe} \\ 0.420, & \text{steel pipe} \end{cases}$$

and A accounts for the effect of pipe roughness (smooth and commercial steel). The term C'_{ij} is the derivative of the cost function per unit length with respect to D/D_u . For the two-part cost model from above, obtain

$$C'_{ij} = \begin{cases} a_s b_s \left(\frac{D_{ij}}{D_u}\right)^{b_s-1}, & D_{ij} \leq D_{co} \\ c_2 + 2c_3 \left(\frac{D_{ij}}{D_u}\right) + 3c_4 \left(\frac{D_{ij}}{D_u}\right)^2, & D_{co} < D_{ij} < D_{co+1} \\ a_\ell b_\ell \left(\frac{D_{ij}}{D_u}\right)^{b_\ell-1}, & D_{ij} \geq D_{co+1} \end{cases} \quad (21)$$

Equation (18), and its simpler form Eq. (15), forms the basis for calculus-based optimization in this work and is applied at all internal nodes to uniquely determine h_j . Equation (18) is valid over the range of $\sim 4000 < \text{Re} < \sim 300,000$.

5 Algorithms to solve a general set of independent, nonlinear algebraic equations using, for example, the Levenberg-Marquardt, Quasi-Newton, Newton-Raphson, or Conjugate Gradient methods are available in most commercial math packages including Matlab (1 Apple Hill Drive, Natick, MA USA 01760) and Mathcad (<http://www.ptc.com>). We used the package Mathcad in the present work. Thus, compared with an iterative solution procedure, a solution flowchart is not relevant here.

10 Bhave (1978) first proposed an algorithm like Eq. (15). However, Bhave used an iterative method to solve the design problem. As such, there are several qualifications leading up to the cost minimization equation in Bhave. These include the assumption of smallness in variation of the static pressure head between two iterations. This allowed the terms in the cost function to be approximated as constants. In the present work, the cost-function coefficient and exponent are not assumed constant at any node joining two sets of links; see Equation (18). Nor do we make any assumptions on the orders of magnitude of the terms in our equations to simplify them. For clarity, we re-present Eq. (15) using Bhave's (1978) notation as

$$0 = \sum Q_{ij}^{7b/19} S_{ij}^{-(1+4b/19)} - \sum Q_{jk}^{7b/19} S_{jk}^{-(1+4b/19)}$$

15 where the ij and jk notation are shown in Fig. 4. Index j spans all internal nodes along the distribution main.

4 Backtracking Algorithm and Genetic Algorithm

Backtracking (BT) and genetic algorithm (GA) assess candidate solutions composed of discrete diameters from a commercially available set. These candidates are represented by a vector of size $[1, N_L]$ where each element corresponds to a network link.

20 The values of the vector specify a diameter from the commercially available set that are indexed from smallest ($i_D = 1$) to largest ($i_D = N_D$). To reduce the computational time associated with these evaluations, the constraints imposed by the energy

equation and cost minimization may be more efficiently evaluated through lookup tables. With fixed L , Δz , K , $L_{\text{by}D}$, and α , the change in static pressure head Δh is evaluated for all $N_D \times N_L$ combinations of pipe diameter and link index:

$$\Delta \mathbf{h} = \begin{bmatrix} \Delta h_{11} & \cdots & \Delta h_{1N_L} \\ \vdots & \ddots & \vdots \\ \Delta h_{N_D 1} & \cdots & \Delta h_{N_D N_L} \end{bmatrix} \quad (22)$$

While an algorithm evaluates a candidate solution, the static pressure head at each node is sequentially calculated by “marching” through the network. Starting with the fixed source pressure head, the algorithm finds the pressure head h_i for a given node by adding the head at the upstream node, h_{i-1} to the change in head for that link i_L and the diameter i_D under consideration. Thus,

$$h_i = h_{i-1} + \Delta \mathbf{h}(i_D, i_L) \quad (23)$$

Along with the hydraulic evaluation of a candidate solution, the cost of the partial candidate is found through the use of a lookup table \mathbf{C} ,

$$\mathbf{C} = \begin{bmatrix} C_{11} & \cdots & C_{1N_L} \\ \vdots & \ddots & \vdots \\ C_{N_D 1} & \cdots & C_{N_D N_L} \end{bmatrix} \quad (24)$$

where $\mathbf{C}(i_D, i_L)$ returns the additional cost of assigning a diameter with index i_D to link i_L . In this way, the candidate solution’s hydraulic performance and cost are incorporated into the genetic algorithm and backtracking approaches. In contrast to GA, the backtracking algorithm evaluates static pressure head and cost upon consideration of each partial candidate, where GA calculates these values on full candidates as part of the objective function.

4.1 BT and GA Pre-Processor 1: Maximum Available Diameter

To increase the efficiency of BT and GA, it is advantageous to limit the number of pipe diameters in the available set, especially those outside of the range of the optimal solution. In particular for the BT algorithm, larger diameters can require considerable computational effort, since they tend not to violate static head requirements and require multiple-link partial candidates for the algorithm to reject them once their cost exceeds that of an already-found viable candidate. Therefore, a pre-processor is used to provide a maximum diameter (D_{max}) that should be considered during the optimization process. This procedure, which produces a conservative estimate, finds the smallest diameter at which a network with a single pipe diameter choice produces no nodes with a static pressure head below h_{min} , similar to the technique used by Mohan and Jinesh Babu (2009). After this diameter is found, the next-larger diameter in the set is selected as D_{max} to allow the algorithm to select a larger-than-necessary diameter if this is able to save cost elsewhere. If D_{max} appears in the optimum solution, the designer can elect to further increase this maximum diameter. It worth noting that Kadu et al. (2008) presents another method to further prune the search space with the critical path concept, where Dongre and Gupta (2011) noted the computational advantages of having just four diameter choices per link. This method, however, may prune the global optimum and may not produce feasible head values at intermediate nodes, as in the case of networks with a local high point.

4.2 BT and GA Pre-Processor 2: Adjusted Minimum Static Pressure Head

A second pre-processor adjusts the minimum static pressure head requirement for each internal node by considering the total head required at downstream nodes. It can be recognized that, without the use of a pump, the total head cannot increase at nodes downstream of a given node i . Furthermore, the total head must decline at a minimum grade that is determined by the demand volume flow rate and the largest pipe diameter available (D_{max}) for selection. This energy constraint is utilized to reduce the number of candidates to be considered by increasing the minimum static pressure head at nodes where these rules produce a higher minimum head than the original h_{min} . For example, nodes upstream of a local network high point can have their minimum static pressure head increased beyond the normal minimum, since the static pressure head must be great enough to ensure adequate flow to the higher-elevation downstream node. To begin this process, each node i is initialized with a baseline minimum total head,

$$th_{min,i} = z_i + h_{min} \quad (25)$$

$th_{min,i}$ is thus initialized by considering only the node's hydraulic requirements in isolation, i.e., without acknowledging the neighboring downstream nodes. The pre-processor then considers updating $th_{min,i}$ by checking the following condition, which is false when the minimum static pressure head at downstream nodes produces further constraints on an upstream node i . Thus, for all nodes i which are upstream of some node j , the following inequality can be evaluated

$$th_{min,i} - th_{min,j} \geq \left(\alpha_{i-j} + K_{i-j} + f_{i-j} \left(\frac{L_{i-j}}{D_{i-j}} + L_{ebyD_{i-j}} \right) \right) \frac{8Q_{i-j}^2}{\pi^2 g D_{max}^4} \quad (26)$$

Also, consider that when flow rate Q_{i-j} is small and D_{max} is large, the right hand side of Eq. (26) approaches zero, representing the simple statement that upstream total head must always be greater than downstream total head. When the condition in Eq. (26) is false, the minimum total head can be updated in node i such that the maximum diameter size in link i - j is able to meet the downstream node's minimum total head, or

$$th_{min,i} = th_{min,j} + \left(\alpha_{i-j} + K_{i-j} + f_{i-j} \left(\frac{L_{i-j}}{D_{i-j}} + L_{ebyD_{i-j}} \right) \right) \frac{8Q_{i-j}^2}{\pi^2 g D_{max}^4} \quad (27)$$

In this way, $th_{min,i}$ may be updated for each node until the condition in Eq. (26) is true for all nodes i with a downstream node j connected by a single link.

After the values for $th_{min,i}$ are updated, they are converted back into minimum static pressure head values by subtracting the elevation z_i from $th_{min,i}$. This pre-processor serves to narrow the search for viable candidate solutions by potentially increasing the minimum static pressure head. Since backtracking and GA consider network links in the downstream direction, these algorithms are otherwise blind to future downstream static pressure head requirements. This limitation is alleviated by the pre-processor, which allows these algorithms some implicit information about what local diameter choices

will be viable for the full network solution. Note that both pre-processors discussed will not prune the global optimum from the solution.

4.3 Backtracking Algorithm (BT)

5 The backtracking algorithm employs a systematic search of candidate solutions to find a global optimum. The algorithm works recursively to incrementally build candidate solutions while checking the candidates for hydraulic and cost acceptability. The strength of the BT is that, upon discovery of an infeasible partial candidate, all extensions of that candidate can be eliminated from consideration. In this way, many solutions can be pruned from the solution tree to achieve greater computational efficiency.

10 Two backtracking methods can be found in the literature, namely those by Gessler (1985) and González-Cebollada et al. (2011). The algorithm proposed by Gessler, however, also proposes a pipe-grouping criteria that risks pruning the global optimum and represents a tricky optimization problem in and of itself (Raad 2011). The González-Cebollada algorithm, on the other hand, does not include such criteria to potentially prune a global optimum from consideration, but it halts its search after finding the first feasible solution. In contrast, the BT algorithm in the present study guarantees a global optimum by continuing its search of the solution tree even after the first solution has been found. In addition, the present BT algorithm utilizes Pre-
15 Processors 1 and 2 to further reduce the search space, without risk of pruning the global optimum, in advance of its search routine. Thus, out of the two reported backtracking algorithms in the literature, both do not guarantee a global optimum, while the BT algorithm presented in this work does. It should be noted that BT is known to scale poorly with large network sizes and would not be appropriate for use on large urban networks, though its appropriateness is demonstrated here for GDWNs, given that the test cases used in this paper representative of the sizes of GDWNs that would be expected in practice.

20 Backtracking is a type of partial enumeration method, which Raad (2011) notes can drastically reduce the number of solutions to be evaluated based on two rejection criteria. The first rejection criterion is that when a candidate violates static pressure head constraints, all candidates with equal or lesser diameter sizes can be discarded. This condition is leveraged even more effectively with Pre-Processor 2 above, which can increase static pressure heads at individual nodes by anticipating the head requirements at surrounding nodes. The second rejection criterion is that once a feasible candidate has been found, all
25 other partial candidates with a higher cost can also be discarded. The BT algorithm further extends this criterion by considering that the links yet to be considered in a partial candidate, an “extension” to the partial candidate, will cost at a minimum that of the entire extension being composed of the smallest available diameter. Thus, when considering whether the partial candidate will necessarily be more expensive than the running optimum, this minimum extension cost can be added to the partial candidate cost.

30 The backtracking algorithm begins its search of the solution tree by considering the partial candidate with the smallest diameter size assigned to the first network link. The static pressure head and the partial candidate cost at the outlet node are calculated with the Δh and C lookup tables. If this partial candidate meets static pressure head and cost requirements, the

algorithm extends this partial candidate by assigning the smallest diameter to the downstream link. If a partial candidate produces a node that is rejected on the basis of static pressure head, the next largest larger diameter is chosen for the link upstream of the node. If no diameter satisfies the pressure head condition, the algorithm backtracks to the upstream link and assigns a larger diameter to the link. If a node is connected to a delivery node by a single link, the smallest feasible diameter for that link is found, and if no such diameter exists, the partial candidate is rejected. In this way, the algorithm continues to extend and reject candidate solutions until a full candidate satisfies the static pressure head requirements. Once this has been achieved, the diameter choices and cost of the network are stored as a running optimum.

Once a working solution has been found, candidate solutions may be rejected based on cost. For each new candidate, cost is calculated by adding the cost of diameters that have already been assigned to the cost of assigning all downstream links with the smallest diameter available. If this cost exceeds the cost of the running optimum, the partial candidate is rejected. Unlike a candidate rejection based on static pressure head, a rejection based on cost does not consider siblings with larger diameters, since these would only further add to cost, rather, the algorithm backtracks immediately to re-assigning the upstream link.

In this way, the rejection criteria based on a minimum static pressure head and cost are used to prune the solution tree and largely reduce the number of non-optimal candidates that need to be considered. The minimum static pressure head criterion tends to prune candidates with diameters that are too small, while the running optimal cost criterion tends to prune candidates of diameters that are too large.

Another pruning technique noted by Raad (2011) is to group together adjacent links that are sized identically. This technique, in contrast to the former two mentioned, cannot guarantee an optimal result, and is therefore not included in the present study's BT algorithm. The present study's BT algorithm operates similarly to the method presented by González-Cebollada et al. (2011), with the major difference being that the BT algorithm continues searching once it has found its first feasible solution and its use of Pre-Processors 1 and 2. The BT algorithm could also be used in this way to find an initial solution very quickly, and then continue as normal to find progressively better solutions until the end of the search space, or a predefined condition such as calculation time, are met.

25 **4.4 Modified Backtracking Algorithm (BT-NoUp)**

A modification to the BT algorithm was made to further improve its computational speed, although at the risk of pruning the global optimum from the search. This modified algorithm (BT-NoUp) rejects all candidates that feature a smaller diameter that is upstream of a larger diameter when an equal or smaller flow rate is present in the downstream link. Typically, optimal networks would not exhibit this feature, and in cases where a single source feeds into a network with constant-length links, it is advantageous (or equivalent) to place larger diameters upstream of smaller diameters. However, due to the discrete nature of diameter choices and link lengths, an optimization problem may, in fact, have an optimal candidate with a larger diameter

downstream from smaller ones. For this reason, the BT-NoUp algorithm, unlike the BT algorithm, may miss the global optimum at the expense of its greater computational efficiency.

4.5 Genetic Algorithm (GA)

5 Genetic algorithms are stochastic optimization techniques that mimic the process of natural selection, and numerous recent variations of GAs have demonstrated improved performance on WDN design (Nicklow et al. 2010). Given their popularity, the GA included in this study is meant to provide a point of comparison to the BT and CB algorithms when applied to GDWNs, and represents a straightforward GA implementation with an attempt to select appropriate operators and well-tuned parameters.

10 When implemented in water network design, each candidate solution represents a selection of pipe diameters. The algorithm is initialized with a population of candidates of size N_c that repeatedly undergoes the processes of mutation, crossover, and selection

$$c_i = [D_{1,i} \quad D_{2,i} \quad \dots \quad D_{N_L,i}] \quad (28)$$

where each candidate in the population c_i contains N_L diameters. In the present work, candidates are represented as a string of natural numbers, which is used over a binary representation to improve the efficiency of encoding and ensure uniqueness of coded strings (Vairavamoorthy and Ali 2000). The mutation operator replaces pipe diameters with a diameter from a uniform random distribution, where each link diameter has a probability of p_{mut} of mutating on each generation. The crossover operator randomly pairs all individuals in the population and performs a single-point crossover of the two individuals with probability p_{xover} , where the point of crossover is chosen randomly in the string of diameters. While a two-point crossover technique was considered, the results were not found to have any benefit over a single-point technique, which was chosen for its greater simplicity. The fitness, f_i , of each candidate is assessed with penalties associated with the solution's pipe cost, $C_{pipe,i}$, and violations of the static pressure head requirement, $C_{hyd,i}$, or

$$f_i = \frac{1}{C_{pipe,i} + C_{hyd,i}} \quad (29)$$

The hydraulic cost is obtained for each individual by identifying nodes in which the static pressure head is less than h_{min} and multiplying the total amount of head violation by a hydraulic penalty coefficient, a_{hyd} :

$$C_{hyd,i_c} = a_{hyd} \sum_1^{N_L} (h_{min} - h_{i_N}) \mid h_{i_N} < h_{min} \quad (30)$$

To allow for a hydraulic penalty coefficient to produce similar results in both small-scale (inexpensive) network and a large-scale (more expensive) cases, the hydraulic penalty coefficient is made directly proportional to the average solution

cost. With each generation, a_{hyd} is updated by multiplying the normalized penalty coefficient, $a_{hyd,norm}$, by the average pipe cost of the population,

$$a_{hyd} = a_{hyd,norm} \frac{\sum_1^{N_c} C_{pipe,iC}}{N_c} \quad (31)$$

The algorithm then selects candidates to be carried into the next generation through a proportionate selection method, where each candidate has a probability of being selected, $p_{sel,i}$ in direct proportion to its fitness relative to the sum of all fitness values in the population

$$p_{sel,i} = \frac{f_i}{\sum_1^{N_c} f_i} \quad (32)$$

The algorithm replaces the parent generation with a generation of equal size and tends to select more fit individuals in successive generations. In this study, the genetic algorithm parameters used were $p_{mut} = 0.02$, $N_c = 50$, $p_{xover} = 0.5$, $N_{gen} = 100$, $a_{hyd,norm} = 0.1$. The first four of these parameters were chosen based on typical values presented in the literature and then tuned with a sensitivity analysis for the first test case. Simson et al. (1994) present typical values for N_c (30 - 200), p_{xover} (0.7 - 1.0), p_m (0.01 - 0.05), and N_{gen} (100 - 1000). The normalized hydraulic penalty coefficient, $a_{hyd,norm}$, was chosen such that the GA converged on solutions which tended to satisfy the minimum static pressure constraint, but still allowed the population to gravitate towards smaller diameters with static pressures close to h_{min} .

5 Cases Studied

Five cases were studied based on actual GDWN in Panama, Nicaragua, and the Philippines. Global characteristics of each network are presented in Table 1 and the details of each network are presented in Table 3(a)-(e). Each network is a branching type without loops. The total lengths of the networks range from less than 1 km to over 15 km. Two serial networks are tested to demonstrate the effect of a local high point on the algorithm solutions. Elevation plots for each case are shown in Fig. 5.

The choice of h_{min} is not standardized, and should appropriately balance the risk of negative pressure in pipes and the increase in network cost due to the requirement of using larger diameters. The choice of h_{min} in GDWN design is typically in the range of 5 m – 20 m (Arnalich 2010; Bouman 2014; Swamee and Sharma 2008). In the present study $h_{min} = 7$ m, although this requirement was reduced at selected nodes at the beginning of networks where changes in elevation are still small. At the source node, the static pressure head is fixed at atmospheric pressure. All cases assumed minor-lossless flow, although all algorithms (e.g., Eq. (18) for CB-Theor) are capable of handling minor loss coefficients through the equivalent length method as presented above.

6 Mapping the Theoretical D to Discrete Pipe Sizes

The mapping between continuous diameters and the discrete nominal pipe sizes was accomplished in our solution by one of the following ways:

1. For small and moderate size networks, the designer may manually adjust the pipe sizes (downward, normally one pipe size) starting from the first link downstream from the source and continuing along the rest of the distribution main to the end. A nearby plot of the static pressure heads compared with the theoretical D_{ij} from our CB approach (on the same Mathcad page) will highlight the acceptability or unacceptability of any change. This exercise also gives the designer an understanding of the sensitivity of the design to small changes in pipe sizes.
2. Based on the theoretical D_{ij} from the CB approach, a composite pipeline can be created for each link. That is, the lengths for the two discrete pipe sizes that bound the theoretical D_{ij} from above and below are calculated such that the pressure drop between two consecutive nodes in the distribution main matches between the composite pipeline and the CB approach. This also provides discrete pipe sizes that nearly matches the CB solution in terms of cost.

7 Results

The current study evaluated three types of algorithms that optimize the design of gravity-driven water networks (GDWN). The algorithms include a calculus-based (CB) algorithm, a backtracking algorithm (BT), a modified backtracking version (BT-NoUp), and a genetic algorithm (GA). The algorithms were applied to five test cases that are based on real GDWNs.

The global optimum network cost, found with BT, is shown in Table 2. The costs of solutions from all other algorithms are expressed as a percentage difference in cost from the global optimum cost. To visually compare the algorithm solutions, the hydraulic grade lines from BT, BT-NoUp, CB-Theor, and CB-Disc are presented in Fig. 5 along with the network elevation for each test case. For clarity, the hydraulic grade lines of branch links are omitted from the figure. In addition, the GA solutions are omitted since 100 solutions were obtained for each test case. Collectively, the hydraulic grade lines reveal a close alignment of the BT solution (the global optimum) with the CB-Theor solution which utilizes a continuous diameter set. Furthermore, the mapping scheme used to generate a CB-Disc solution is shown to increase pipe sizes in some cases far beyond the limit imposed by h_{min} , which was set to 7 m in the present work.

In practice, a GDWN must be designed with pipe diameters that are selected from a discrete, commercially available set. With a given number of network links, N_L , and a number of available diameters, N_D , a total of $N_D^{N_L}$ candidate solutions exist, yet with only one global optimum (except in the case of no viable solutions or unique solutions with identical costs). For example, a GDWN of 20 links and 13 commercially available pipe sizes will, in principal, produce approximately 1.9×10^{22} candidate solutions.

BT is able to find the global optimum without needing to check all of the possible solutions by using a set of rules to prune infeasible and sub-optimal candidates. In this study, BT evaluated only a fraction of the candidate solutions, where the fraction ranged from 4×10^{-18} to 7×10^{-4} . To further reduce the number of evaluations required to arrive at a solution, the BT algorithm was modified (BT-NoUp) to prune all solutions that include a smaller diameter that is upstream of a larger diameter. This criterion, which seems intuitive to the designer, may actually miss the global optimum due to trade-offs associated with discrete solutions. In fact, BT-NoUp missed the global optimum in cases 2 and 3, although by a small percentage increase in cost (2.1% and 0.35% respectively). BT-NoUp, however, finished its search in a shorter amount of time in comparison to BT. Using a Dell Latitude (i5 CPU at 2.50 GHz), the time to evaluate one candidate for both BT and BT-NoUp was around 0.2 ms. In addition to approximately 2 s of pre-processing time, the computation times for BT ranged from 0.08 s (case 1) to 79 s (case 3), while BT-NoUp ranged from 0.06 s (case 4) to 0.3 s (case 3). The number of available diameters used in the BT, BT-NoUp, and GA runs are listed in Table 1.

The CB algorithm, unlike the other algorithms in this work, finds a solution with theoretical diameters that are drawn from a continuous domain (CB-Theor). For all test cases, the costs of the CB-Theor solutions was less when compared with the discrete-diameter global optimum (-5.46% to -2.60%). In fact, because of the discrete pipe sizes needed for an actual network, *the continuous model will always produce the smallest theoretical cost for the network*. The CB algorithm then maps this solution to a commercially-available discrete set (CB-Disc). The mapping process used in this study simply mapped each theoretical diameter to the nearest available diameter of a larger size, thus producing a solution which still satisfies static head requirements but with a higher associated material cost. This tended to oversize the diameters, although the CB-Disc solutions were always within two diameters of the BT global optimum solutions, as shown in Fig. 6. From all of the test cases combined, all but one (71 out of 72) of the diameter selections were within one diameter of the global optimum. More sophisticated mapping schemes, like independently adjusting D for each link in the distribution main in a step-by-step manner starting with the source while ensuring all pressure head constraints are satisfied, would more likely produce results identical to the global optimum. This was not performed in the current study. The CB-Disc solution costs were, in all cases, larger than the global optimum, with a percentage difference ranging from 3.86% to 22.6%. Thus, for all cases, the calculus-based algorithm bounded the cost of the global optima with a lower-cost CB-Theor solution and a higher-cost CB-Disc solution. This trend is a result of the additional constraints imposed by the finite set of diameter choices. If the algorithm is allowed a greater number of discrete diameter choices, i.e., through adding a less-common nominal diameter size to the available set, the cost of the CB-Disc solution would approach the CB-Theor solution. For all cases, the CB algorithm converged on a solution in 5 minutes or less.

GA was run on each case a total of 100 times, each run itself produced 100 generations of 50 candidates. The least-cost candidate in that did not violate the static pressure head condition was chosen as the optimum. Because GA is a stochastic search algorithm producing different results from run-to-run, the costs of the optima from all 100 runs were averaged, with this averaged value presented in Table 2 as a percentage increase from the global optimum. Out of the 100 GA runs for each test case, nearly all runs failed to achieve the global optimum, with the exception of 8 runs of the Kiangnan network. On a Dell

Latitude (i5 CPU at 2.50 GHz), GA runs took between 0.9 s (case 1) and 1.2 s (case 3), not including about 2 s of pre-processor time. We note that many variations of GAs have been reported in the literature and several of these would likely improve upon the GA results obtained in this study. Potential improvements to the GA a self-adapting penalty function (Wu and Walski 2005), the use of elitism to preserve the best solutions (Kadu et al. 2008), and a reduction in the search space (Kadu et al. 2008). Other reported improvements, including the systematic optimization of operator parameters (Reed et al. 2000) and scaling of the fitness function to magnify the rewards towards slightly fitter candidates (Dandy et al. 1996) were attempted in a less systematic way (I.e. a parameter tuning study and three attempts at scaling the fitness function with an increasing exponent), but these did not result in a noticeable on performance. However, it is possible that if these techniques were followed systematically in full, the GA performance may have been improved. Still, the GA algorithm presented has undergone reasonable attempts to adapt its design and parameters for real-world GDWN cases, and therefore presents a useful point of comparison to the BT and CB algorithms.

8 Conclusions

Algorithms to optimize the cost of branching gravity-driven water networks are evaluated on five test cases from real networks in the Philippines, Nicaragua, and Panama. A calculus-based algorithm produced a solution composed of theoretical diameters from a continuous set (CB-Theor), which are then mapped onto discrete commercially available diameters (CB-Disc). Backtracking (BT), a recursive algorithm, systematically searches discrete candidate solutions and is guaranteed to find the global optimum by following rules that prune only higher-cost or hydraulically infeasible candidates. The BT algorithm was modified (BT-NoUp) to improve computational speed by also rejecting all candidates that included a small diameter directly upstream of a larger diameter. This criterion allowed BT-NoUp to prune more candidate solutions but allowed for the possibility of missing the global optimum. The third type of algorithm evaluated was a genetic algorithm (GA) that used single-point crossover and proportionate selection

BT was able to find the global optimum in all test cases with relatively little computational effort, and could be applied to other GDWNs composed of a similar number of links. In addition, while BT-NoUp completed its search in less time than BT, the time required to complete BT would not be burdensome on a designer and therefore BT-NoUp did not produce a compelling relative advantage over BT. BT, however, could become prohibitively time-consuming when dealing with networks with significantly more links, as would be the case with large urban networks. While the test cases represent the range of GDWN sizes encountered in the authors' experience, future work would be needed to verify the suitability of the BT and BT-NoUp algorithms on other large GDWNs. The calculus-based algorithm produced consistently good results for the networks tested, although a more robust mapping scheme from theoretical diameters to discrete diameters would further improve on these results as discussed above. In potential future work, the CB-Theor solutions could be used to prune the BT search space, similar to Kadu et al. (2008), by only including the two diameters above and below the CB-Theor diameters,

producing four diameter choices per link. The calculus-based methodology provides an additional benefit to the designer by explicitly revealing the sensitivities to cost for a design. The calculus-based algorithm requires greater computational effort than backtracking for smaller networks, however, this effort scales more linearly with the number of network links, while backtracking scales exponentially. Furthermore, backtracking's computational time is sensitive to the number of available diameters. Still, when applied to GDWNs with a similar number of links to the test cases, backtracking can quickly find a global optimum. In addition, because it is guaranteed to find the global optimum, it can be useful for benchmarking the performance of other algorithms which scale better with more network links. While the genetic algorithm produced solutions with decent closeness to the global optimum, run-to-run results vary due to the stochastic nature of the algorithm. Overall, the genetic algorithm as implemented did not produce results which deemed it compelling over deterministic methods as applied to GDWNs. However, for more complex networks and problem formulations, a genetic algorithm may be more advantageous. In this case, the present study's GA could be greatly improved on through many improvements reported in the literature (Nicklow et al. 2010).

For all test cases, the calculus-based algorithm's theoretical diameter solutions (CB-Theor) produced a lower cost than the discrete-domain global optimum. This result is made possible because of it is not constrained to a discrete set of diameters. As such, the *CB-Theor results represent a lower-bound on the optimum solution within the problem formulation*, which could be approached with a finer selection of pipe diameters.

9 References

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Table 1: Characteristics of Test Cases.

Test Case	Type	Number of Diameter Choices	Number of Links	Q_{tot} (L s ⁻¹)	L_{tot} (km)
1. Kiagan, Philippines	Branching	8	9	4.37	0.82
2. Los Modulus, Nicaragua	Serial	4	13	0.39	1.24
3. Cañazas, Panama	Branching	10	23	6.29	15.2
4. San Miguel, Nicaragua	Serial	9	10	0.40	1.18
5. El Guabo, Nicaragua	Branching	12	17	17.7	4.71

Table 2: Solution Costs.

Test Case	Global Optimum	Percentage cost increase from global optimum			
	BT	BT-NoUp	CB-Theor	CB-Disc	GA
1. Kiagan, Philippines	\$ 2,331	0	-3.16	11.3	4.80
2. Los Modulos, Nicaragua	\$ 1,441	2.10	-2.60	22.6	12.1
3. Cañazas, Panama	\$ 72,190	0.35	-5.46	17.0	20.7
4. San Miguel, Nicaragua	\$ 5,418	0	-4.54	3.86	6.20
5. El Guabo, Nicaragua	\$ 61,445	0	-3.16	20.2	13.3

Table 3(a): Case #1 Kiangnan network properties, diameter (D) results (inch nominal sizes, with CB-Theor in inches), and nodal h.

Network	Link	1-2	2-3	3-4	4-5	5-6	2-7	3-8	4-9	5-10	
	Length (m)	76	113	19	54	75	80	99	170	135	
	Q (L/s)	4.37	3.68	2.94	1.46	0.69	0.69	0.74	1.48	0.77	
	Δz (m)	14.0	1.0	0.0	0.0	-1.0	0.0	-2.0	3.0	2.0	
D solutions	BT	3	2-½	2	1-½	1-½	1	1-¼	1-½	1-¼	
	BT-NoUp	3	2-½	2	1-½	1-½	1	1-¼	1-½	1-¼	
	CB-Theor	2.751	2.562	2.141	1.830	1.356	1.062	1.376	1.584	1.128	
	CB-Disc	3	3	2-½	2	1-¼	1-¼	1-¼	1-½	1-¼	
h (m)	Node	1	2	3	4	5	6	7	8	9	10
	BT	0	13.09	11.43	10.72	8.81	7.10	7.27	7.21	7.57	7.58
	BT-NoUp	0	13.09	11.43	10.72	8.81	7.10	7.27	7.21	7.57	7.58
	CB-Theor	0	12.48	11.24	10.65	9.61	7.00	6.99	7.00	7.00	3.19
	CB-Disc	0	13.09	13.15	12.85	12.27	9.78	11.51	8.94	9.70	11.04

Table 3(b): Case #2 Los Modulos network properties, diameter (D) results (inch nominal sizes, with CB-Theor in inches), and nodal h.

Network	Link	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12	12-13	13-14	
	Length (m)	60	41	108	46	134	153	79	157	90	32	102	120	117	
	Q (L/s)	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39	
	Δz (m)	11.2	-0.5	32.8	-3.7	36.6	-2.3	15.7	-6.8	7.3	-7.4	4.5	-1.2	8.4	
D solutions	BT	1	1	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1	$\frac{3}{4}$	1	1	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	
	BT-NoUp	1	1	1	1	1	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	
	CB-Theor	0.987	0.984	0.849	0.849	0.849	0.849	0.849	0.849	0.849	0.849	0.849	0.849	0.849	
	CB-Disc	1	1	1	1	1	1	1	1	1	1	1	1	1	
h (m)	Node	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	BT	0	9.55	7.94	31.59	24.00	49.25	33.99	47.55	27.45	32.32	24.05	19.91	8.55	7.04
	BT-NoUp	0	9.55	7.94	37.82	32.88	65.85	50.59	59.60	39.50	39.18	29.07	24.93	13.56	12.05
	CB-Theor	0	9.00	7.00	31.86	24.78	51.53	37.98	47.87	29.53	30.21	20.46	17.46	7.44	7.23
	CB-Disc	0	9.55	7.94	37.82	32.88	65.85	59.41	72.98	61.93	66.80	58.53	60.27	55.83	61.06

Table 3(c): Case #3 Cañazas network properties, diameter (D) results (inch nominal sizes, with CB-Theor in inches), and nodal h.

Network	Link	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12	12-13	2-14	3-15	4-16	5-17	6-18	7-19	8-20	9-21	10-22	11-23	12-24	
	Length (m)	646	275	957	509	1102	291	1764	1256	2320	1580	2170	1217	160	100	1250	110	570	180	1400	50	400	260	100	
	Q (L/s)	6.29	5.49	5.39	5.34	5.14	2.84	2.74	2.49	2.39	0.69	0.39	0.20	0.80	0.10	0.05	0.20	2.30	0.10	0.25	0.10	1.70	0.30	0.19	
	Δz (m)	25.0	38.9	11.9	42.1	-22.9	32.3	-29.9	40.8	-3.0	-14.7	34.1	-7.6	-5.0	20.0	-15.0	2.0	-12.0	14.0	-6.0	5.0	-1.0	-13.0	9.0	
D solutions	BT	4	3	3	4	3	3	3	2-½	2-½	2	1-¼	1	1-¼	½	½	½	2	½	1	½	1-½	1-¼	½	
	BT-NoUp	4	4	4	4	3	3	2-½	2-½	2-½	2	1-¼	1	1-¼	½	½	½	1-½	½	1	½	1-½	1-½	½	
	CB-Theor	3.530	3.531	3.333	3.307	3.270	2.727	2.698	2.579	2.548	1.862	1.227	1.011	1.283	0.325	0.508	0.404	1.678	0.343	0.963	0.281	1.405	1.401	0.488	
	CB-Disc	4	4	4	4	4	3	3	3	3	2	1-¼	1	1-¼	½	½	½	2	½	1	½	1-½	1-½	½	
h (m)	Node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
	BT	0	21.1	55.4	51.5	91.3	51.7	82.5	43.9	69.8	41.4	22.1	40.0	21.7	12.1	72.2	24.3	82.2	26.1	90.8	20.1	73.3	21.9	7.81	39.7
	BT-NoUp	0	21.1	58.8	66.4	106	66.6	97.4	42.8	68.8	40.4	21.0	39.0	20.7	12.1	75.6	39.2	97.1	9.5	106	19.0	72.2	20.9	7.43	38.7
	CB-Theor	0	17.8	54.3	55.6	91.9	56.7	86.3	40.3	69.0	44.1	21.9	27.9	7.64	6.99	8.02	7.70	7.98	7.72	7.79	7.70	8.18	7.68	7.69	7.71
CB-Disc	0	21.1	58.8	66.4	106	78.8	110	70.9	106	94.4	75.1	93.0	74.7	12.1	75.6	39.2	97.1	53.1	118	47.1	110	74.9	61.4	92.7	

Table 3(d): Case #4 San Miguel network properties, diameter (D) results (inch nominal sizes, with CB-Theor in inches), and nodal h.

Network	Link	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	
	Length (m)	189	168	139	81	32	92	225	115	52.3	85	
	Q (L/s)	3.60	3.60	3.60	3.60	3.60	3.60	3.60	3.60	3.60	3.60	
	Δz (m)	27.4	10.7	-6.4	6.1	-5.2	-18.6	33.2	58.2	-11.3	32.9	
D solutions	BT	3	3	3	3	3	2-½	2	1-¼	1-¼	1-¼	
	BT-NoUp	3	3	3	3	3	2-½	2	1-¼	1-¼	1-¼	
	CB-Theor	2.939	2.929	2.929	2.929	2.929	2.929	1.671	1.462	1.462	1.368	
	CB-Disc	3	3	3	3	3	3	2	1-½	1-½	1-¼	
h (m)	Node	1	2	3	4	5	6	7	8	9	10	11
	BT	0	25.88	35.20	27.68	33.13	27.70	7.02	28.27	43.86	13.19	14.60
	BT-NoUp	0	25.88	35.20	27.68	33.13	27.70	7.02	28.27	43.86	13.19	14.60
	CB-Theor	0	25.53	34.51	26.72	32.01	26.51	7.00	6.99	32.93	6.96	7.02
	CB-Disc	0	25.88	35.20	27.68	33.13	27.70	8.37	29.62	67.54	47.02	48.43

Table 3(e): Case #5 El Guabo network properties, diameter (D) results (inch nominal sizes, with CB-Theor in inches), and nodal h.

Network	Link	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	2-11	3-12	4-13	5-14	6-15	7-16	8-17	9-18	
	Length (m)	383	486	1030	600	150	400	187	450	227	230	240	110	270	130	130	260	110	
	Q (L/s)	17.72	14.68	12.76	11.96	10.04	7.72	6.60	3.12	1.20	3.04	1.92	0.80	1.92	2.32	1.12	3.48	1.92	
	Δz (m)	10.9	10.0	-5.6	3.2	-2.6	5.7	-4.1	4.2	-3.1	2.0	2.5	-1.2	2.0	-1.1	0.0	1.0	2.0	
D solutions	BT	8	6	6	6	6	5	5	4	2	2-½	1-½	1-½	2	3	1-¼	3	1-½	
	BT-NoUp	8	6	6	6	6	5	5	4	2	2-½	1-½	1-½	2	3	1-¼	3	1-½	
	CB-Theor	6.875	6.408	6.144	6.008	5.691	4.800	4.576	3.494	2.649	2.364	1.608	1.529	1.932	3.250	1.395	3.076	1.647	
	CB-Disc	8	8	8	6	6	5	5	4	3	2-½	1-½	1-½	2	4	1-½	4	2	
h (m)	Node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	BT	0	10.34	18.50	9.91	11.53	8.61	13.16	8.65	12.11	7.25	8.48	7.23	7.35	8.84	7.03	7.16	7.68	7.80
	BT-NoUp	0	10.34	18.50	9.91	11.53	8.61	13.16	8.65	12.11	7.25	8.48	7.23	7.35	8.84	7.03	7.16	7.68	7.80
	CB-Theor	0	9.76	18.35	9.93	11.49	8.46	12.70	7.94	10.67	7.00	7.00	7.00	7.00	7.01	7.00	7.00	7.00	7.01
	CB-Disc	0	10.34	19.85	13.47	15.09	12.17	16.72	12.21	15.67	12.27	8.48	8.58	10.91	12.40	10.94	13.84	12.67	15.76

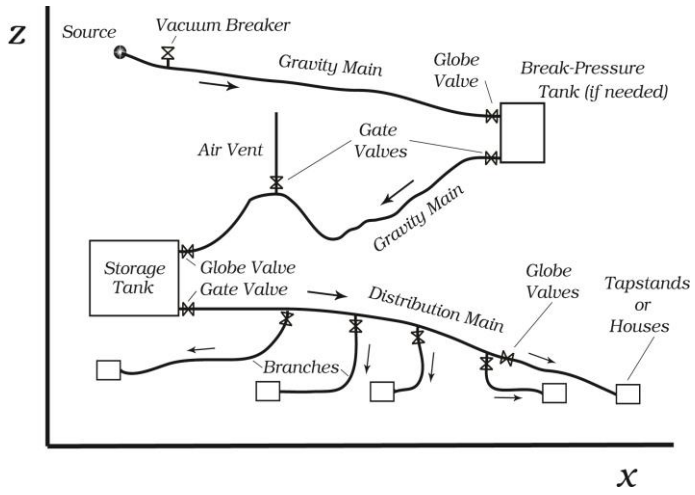


Figure 1: Element schematic of a GDWN.

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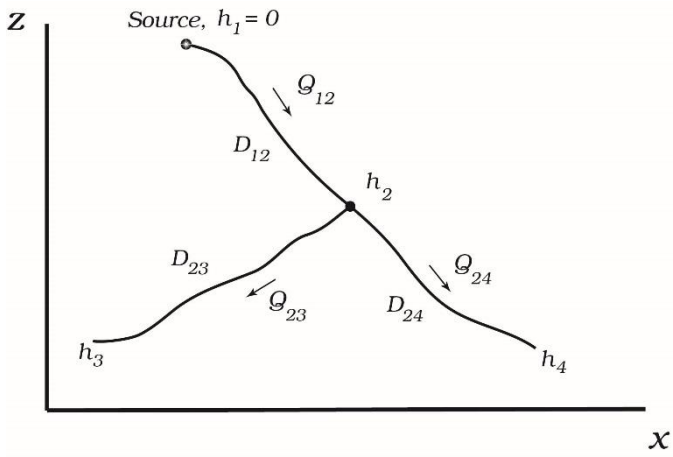


Figure 2: Three-pipe branch network.

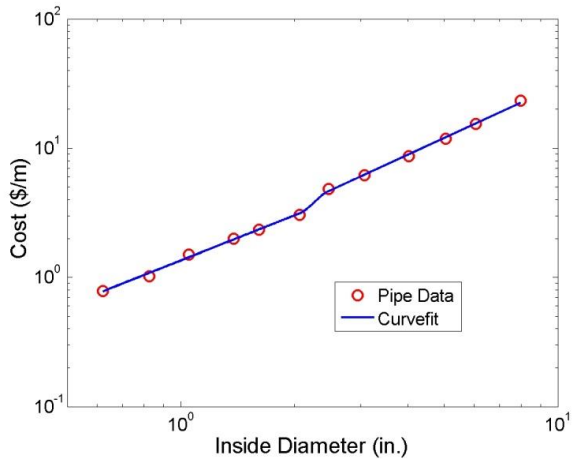


Figure 3: PVC pipe cost from 2011 data.

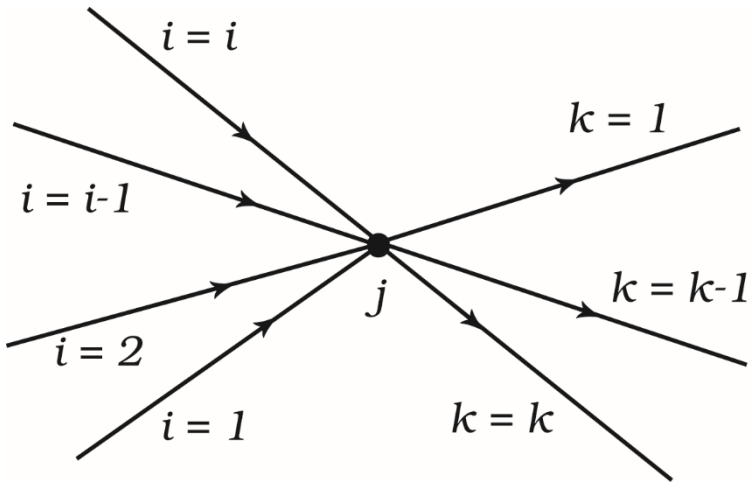


Figure 4. Bhava (1978) index notation at an internal node, j .

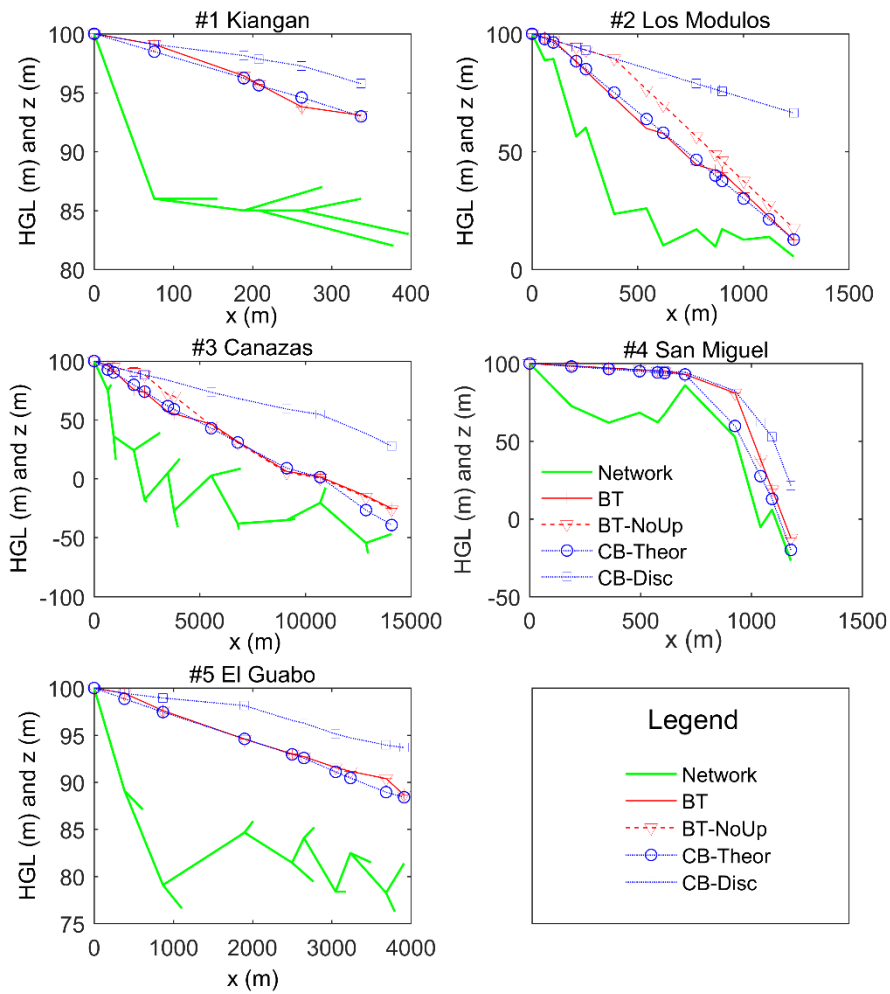


Figure 5: Network elevation (z) and hydraulic grade lines (HGLs) of algorithm solution for main distribution links.

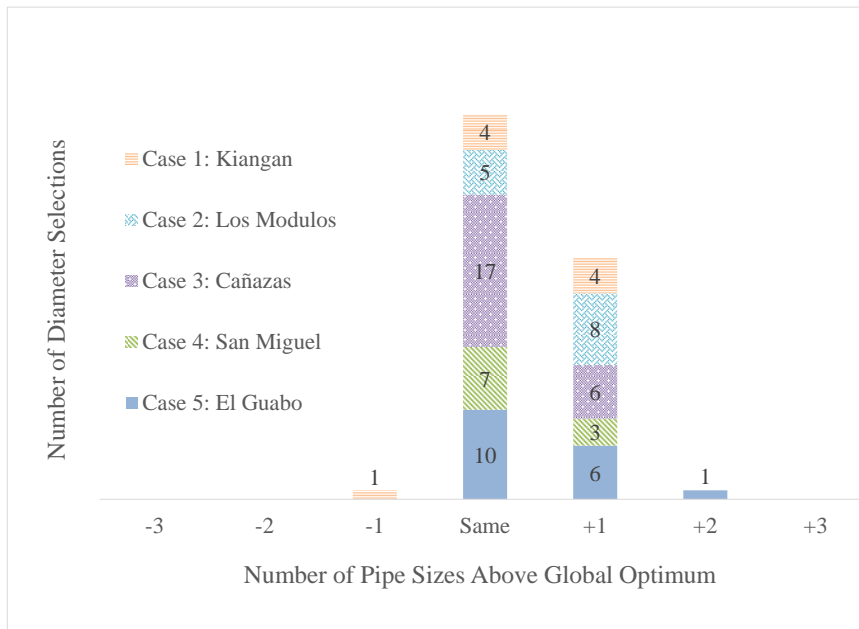


Figure 6: Diameter sizes of calculus-based (CB-Disc) solutions above the global optimum solutions.